MODERN MACHINE SHOP PRACTICE.

Chapter I.—THE TEETH OF GEAR-WHEELS.

A WHEEL that is provided with teeth to mesh, engage, or gear with similar teeth upon another wheel, so that the motion of one may be imparted to the other, is called, in general terms, a gear-wheel.

When the teeth are arranged to be parallel to the wheel-axis, as in Fig 1, the wheel is termed a spur-wheel. In the figure, A

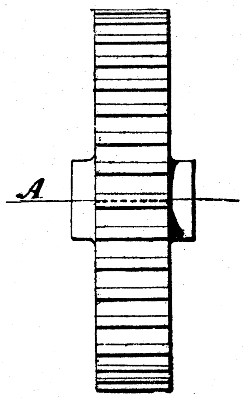

Fig. 1.

represents the axial line or axis of the wheel or of its shaft, to which the teeth are parallel while spaced equidistant around the rim, or face, as it is termed, of the wheel.

When the wheel has its teeth arranged at an angle to the shaft, as in Fig. 2, it is termed a bevel-wheel, or bevel gear; but when

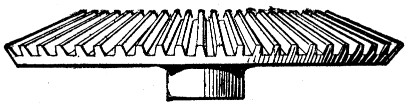

Fig. 2.

this angle is one of 45°, as in Fig. 3, as it must be if the pair of wheels are of the same diameter, so as to make the revolutions of

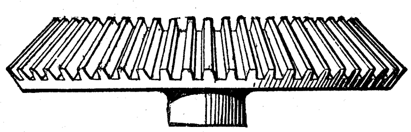

Fig. 3.

their shafts equal, then the wheel is called a mitre-wheel. When the teeth are arranged upon the radial or side face of the wheel,

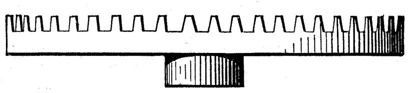

Fig. 4.

as in Fig. 4, it is termed a crown-wheel. The smallest wheel of a pair, or of a train or set of gear-wheels, is termed the pinion; and when the teeth are composed of rungs, as in Fig. 5, it is termed a lantern, trundle, or wallower; and each cylindrical piece serving as a tooth is termed a *stave*, *spindle*, or *round*, and by some a *leaf*.

An annular or internal gear-wheel is one in which the faces of the teeth are within and the flanks without, or outside the pitch-circle, as in Fig. 6; hence the pinion P operates within the wheel.

When the teeth of a wheel are inserted in mortises or slots

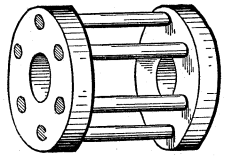

Fig. 5.

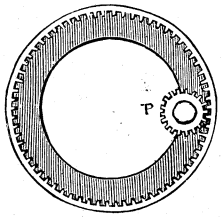

Fig. 6.

provided in the wheel-rim, it is termed a mortised-wheel, or a cogged-wheel, and the teeth are termed cogs.

When the teeth are arranged along a plane surface or straight line, as in Fig. 7, the toothed plane is termed a *rack*, and the wheel is termed a pinion.

A wheel that is driven by a revolving screw, or worm as it is termed, is called a worm-wheel, the arrangement of a worm and worm-wheel being shown in Fig. 8. The screw or worm is sometimes also called an endless screw, because its action upon the wheel does not come to an end as it does when it is revolved in one continuous direction and actuates a nut. So also, since the worm is tangent to the wheel, the arrangement is sometimes called a wheel and tangent screw.

The diameter of a gear-wheel is always taken at the pitch circle, unless otherwise specially stated as "diameter over all," "diameter of addendum," or "diameter at root of teeth," &c., &c.

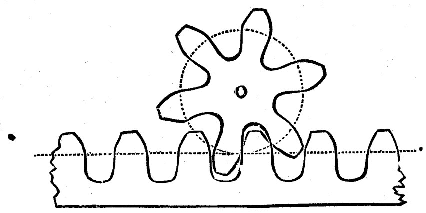

Fig. 7.

When the teeth of wheels engage to the proper distance, which is when the pitch circles meet, they are said to be in gear, or geared together. It is obvious that if two wheels are to be geared together their teeth must be the same distance apart, or the same *pitch*, as it is called.

The designations of the various parts or surfaces of a tooth of a gear-wheel are represented in Fig. 9, in which the surface A is the face of the tooth, while the dimension F is the width of face of the wheel, when its size is referred to. B is the flank or distance from the pitch line to the root of the tooth, and C the point. H is the *space*, or the distance from the side of one tooth to the nearest side of the next tooth, the width of space being measured on the pitch circle P P. E is the depth of the tooth, and G its thickness, the latter also being measured on the pitch circle P P. When spoken of with reference to a tooth,

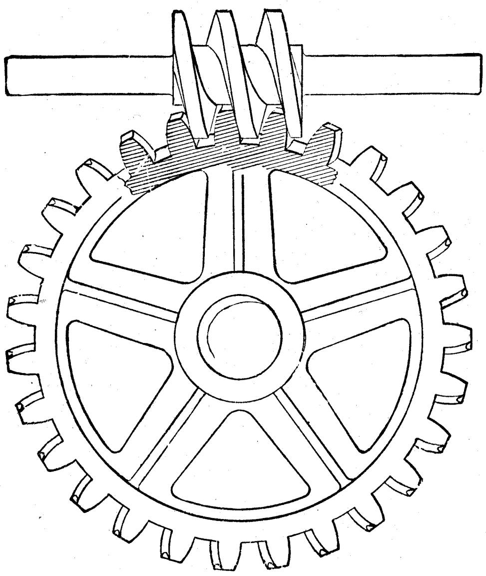

Fig. 8.

P P is called the pitch line, but when the whole wheel is referred to it becomes the pitch circle.

The points C and the surface H are true to the wheel axis.

The teeth are designated for measurement by the pitch; the height or depth above and below pitch line; and the thickness.

The pitch, however, may be measured in two ways, to wit, around the pitch circle A, in Fig. 10, which is called the arc or circular pitch, and across B, which is termed the chord pitch.

In proportion as the diameter of a wheel (having a given pitch) is increased, or as the pitch of the teeth is made finer (on a wheel

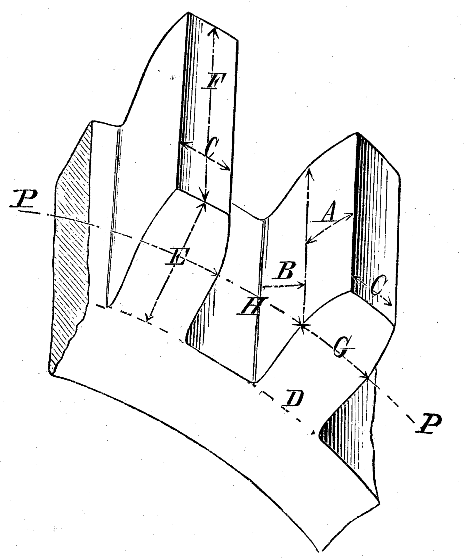

Fig. 9.

of a given diameter) the arc and chord pitches more nearly coincide in length. In the practical operations of marking out the teeth, however, the arc pitch is not necessarily referred to, for if the diameter of the pitch circle be made correct for the required number of teeth having the necessary arc pitch, and the wheel be accurately divided off into the requisite number of divisions with compasses set to the chord pitch, or by means of an index plate, then the arc pitch must necessarily be correct, although not referred to, save in determining the diameter of the wheel at the pitch circle.

The difference between the width of a space and the thickness of the tooth (both being measured on the pitch circle or pitch line) is termed the clearance or side clearance, which is necessary to prevent the teeth of one wheel fıom becoming locked in the spaces of the other. The amount of clearance is, when the teeth are cut to shape in a machine, made just sufficient to prevent contact on one side of the teeth when they are in proper gear (the pitch circles meeting in the line of centres). But when the teeth are cast upon the wheel the clearance is increased to allow for the slight inequalities of tooth shape that is incidental to casting them. The amount of clearance given is varied to suit the method employed to mould the wheels, as will be explained hereafter.

The line of centres is an imaginary line from the centre or axis of one wheel to the axis of the other when the two are in gear; hence each tooth is most deeply engaged, in the space of the other wheel, when it is on the line of centres.

There are three methods of designating the sizes of gear-wheels. First, by their diameters at the pitch circle or pitch diameter and the number of teeth they contain; second, by the number of teeth in the wheel and the pitch of the teeth; and third, by a system known as diametral pitch.

The first is objectionable because it involves a calculation to find the pitch of the teeth; furthermore, if this calculation be

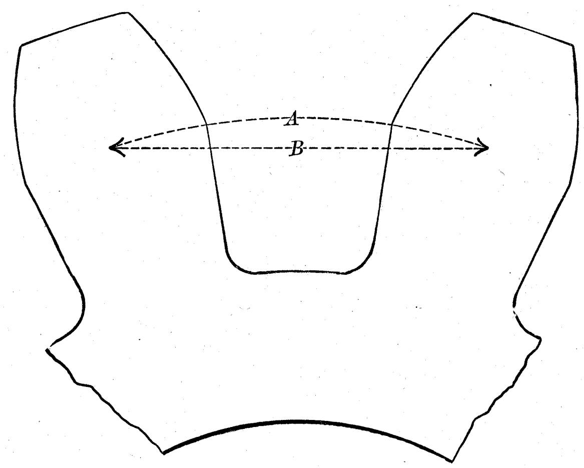

Fig. 10.

made by dividing the circumference of the pitch circle by the number of teeth in the wheel, the result gives the arc pitch, which cannot be measured correctly by a lineal measuring rule, especially if the wheel be a small one having but few teeth, or of coarse pitch, as, in that case, the arc pitch very sensibly differs from the chord pitch, and a second calculation may become necessary to find the chord pitch from the arc pitch.

The second method (the number and pitch of the teeth) possesses the disadvantage that it is necessary to state whether the pitch is the arc or the chord pitch.

If the arc pitch is given it is difficult to measure as before, while if the chord pitch is given it possesses the disadvantage that the diameters of the wheels will not be exactly proportional to the numbers of teeth in the respective wheels. For instance, a wheel with 20 teeth of 2 inch chord pitch is not exactly half the diameter of one of 40 teeth and 2 inch chord pitch.

To find the chord pitch of a wheel take 180 (= half the degrees in a circle) and divide it by the number of teeth in the wheel. In a table of natural sines find the sine for the number so found, which multiply by 2, and then by the radius of the wheel in inches.

Example.—What is the chord pitch of a wheel having 12 teeth and a diameter (at pitch circle) of 8 inches? Here 180 ÷ 12 = 15;

(sine of 15 is ·25881). Then ·25881 × 2 = ·51762 × 4 (= radius of wheel) = 2·07048 inches = chord pitch.

TABLE OF NATURAL SINES.

Degrees.	Sine.	Degrees.	Sine.	Degrees.	Sine.
1	·01745	16	·27563	31	·51503
2	·03489	17	·29237	32	·52991
3	·05233	18	·30901	33	·54463
4	·06975	19	·32556	34	·55919
5	·08715	20	·34202	35	·57357
6	·10452	21	·35836	36	·58778
7	·12186	22	·37460	37	·60181
8	·13917	23	·39073	38	·61566
9	·15643	24	·40673	39	·62932
10	·17364	25	·42261	40	·64278
11	·19080	26	·43837	41	·65605
12	·20791	27	·45399	42	·66913
13	·22495	28	·46947	43	·68199
14	·24192	29	·48480	44	·69465
15	·25881	30	·50000	45	·70710

The principle upon which diametral pitch is based is as follows :—

The diameter of the wheel at the pitch circle is supposed to be divided into as many equal parts or divisions as there are teeth in the wheel, and the length of one of these parts is the diametral pitch. The relationship which the diametral bears to the arc pitch is the same as the diameter to the circumference, hence a diametral pitch which measures 1 inch will accord with an arc pitch of 3·1416 ; and it becomes evident that, for all arc pitches of less than 3·1416 inches, the corresponding diametral pitch must be expressed in fractions of an inch, as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and so on, increasing the denominator until the fraction becomes so small that an arc with which it accords is too fine to be of practical service. The numerators of these fractions being 1, in each case, they are in practice discarded, the denominators only being used, so that, instead of saying diametral pitches of $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$, we say diametral pitches of 2, 3, or 4, meaning that there are 2, 3, or 4 teeth on the wheel for every inch in the diameter of the pitch circle.

Suppose now we are given a diametral pitch of 2. To obtain the corresponding arc pitch we divide 3·1416 (the relation of the circumference to the diameter) by 2 (the diametral pitch), and 3·1416 ÷ 2 = 1·57 = the arc pitch in inches and decimal parts of an inch. The reason of this is plain, because, an arc pitch of 3·1416 inches being represented by a diametral pitch of 1, a diametral pitch of $\frac{1}{2}$ (or 2 as it is called) will be one half of 3·1416. The advantage of discarding the numerator is, then, that we avoid the use of fractions and are readily enabled to find any arc pitch from a given diametral pitch.

Examples.—Given a 5 diametral pitch ; what is the arc pitch ? First (using the full fraction $\frac{1}{5}$) we have $\frac{1}{5}$ × 3·1416 = ·628 = the arc pitch. Second (discarding the numerator), we have 3·1416 ÷ 5 = ·628 = arc pitch. If we are given an arc pitch to find a corresponding diametral pitch we again simply divide 3·1416 by the given arc pitch.

Example.—What is the diametral pitch of a wheel whose arc pitch is $1\frac{1}{2}$ inches ? Here 3·1416 ÷ 1·5 = 2·09 = diametral pitch. The reason of this is also plain, for since the arc pitch is to the diametral pitch as the circumference is to the diameter we have : as 3·1416 is to 1, so is 1·5 to the required diametral pitch ; then 3·1416 × 1 ÷ 1·5 = 2·09 = the required diametral pitch.

To find the number of teeth contained in a wheel when the diameter and diametral pitch is given, multiply the diameter in inches by the diametral pitch. The product is the answer. Thus, how many teeth in a wheel 36 inches diameter and of 3 diametral pitch ? Here 36 × 3 = 108 = the number of teeth sought. Or, per contra, a wheel of 36 inches diameter has 108 teeth. What is the diametral pitch ? 108 ÷ 36 = 3 = the diametral pitch. Thus it will be seen that, for determining the relative sizes of wheels, this system is excellent from its simplicity. It also possesses the advantage that, by adding two parts of the diametral pitch to the pitch diameter, the outside diameter of the wheel or the diameter of the addendum is obtained. For instance, a wheel containing 30 teeth of 10 pitch would be 3 inches diameter on the pitch circle and $3\frac{2}{10}$ outside or total diameter.

Again, a wheel having 40 teeth of 8 diametral pitch would have a pitch circle diameter of 5 inches, because 40 ÷ 8 = 5, and its full diameter would be $5\frac{1}{4}$ inches, because the diametral pitch is $\frac{1}{8}$, and this multiplied by 2 gives $\frac{1}{4}$, which added to the pitch circle diameter of 5 inches makes $5\frac{1}{4}$ inches, which is therefore the diameter of the addendum, or, in other words, the full diameter of the wheel.

Suppose now that a pair of wheels require to have pitch circles of 5 and 8 inches diameter respectively, and that the arc pitch requires to be, say, as near as may be $\frac{4}{10}$ inch ; to find a suitable pitch and the number of teeth by the diametral pitch system we proceed as follows :

In the following table are given various arc pitches, and the corresponding diametral pitch.

Diametral Pitch.	Arc Pitch.	Arc Pitch.	Diametral Pitch.
		Inch.	
2	1·57	1·75	1·79
2·25	1·39	1·5	2·09
2·5	1·25	1·4375	2·18
2·75	1·14	1·375	2·28
3	1·04	1·3125	2·39
3·5	·890	1·25	2·51
4	·785	1·1875	2·65
5	·628	1·125	2·79
6	·523	1·0625	2·96
7	·448	1·0000	3·14
8	·392	0·9375	3·35
9	·350	0·875	3·59
10	·314	0·8125	3·86
11	·280	0·75	4·19
12	·261	0·6875	4·57
14	·224	0·625	5·03
16	·196	0·5625	5·58
18	·174	0·5	6·28
20	·157	0·4375	7·18
22	·143	0·375	8·38
24	·130	0·3125	10·00
26	·120	0·25	12·56

From this table we find that the nearest diametral pitch that will correspond to an arc pitch of $\frac{4}{10}$ inch is a diametral pitch of 8, which equals an arc pitch of ·392, hence we multiply the pitch circles (5 and 8,) by 8, and obtain 40 and 64 as the number of teeth, the arc pitch being ·392 of an inch. To find the number of teeth and pitch by the arc pitch and circumference of the pitch circle, we should require to find the circumference of the pitch circle, and divide this by the nearest arc pitch that would divide the circumference without leaving a remainder, which would entail more calculating than by the diametral pitch system.

The designation of pitch by the diametral pitch system is, however, not applied in practice to coarse pitches, nor to gears in which the teeth are cast upon the wheels, pattern makers generally preferring to make the pitch to some measurement that accords with the divisions of the ordinary measuring rule.

Of two gear-wheels that which impels the other is termed the driver, and that which receives motion from the other is termed the driven wheel or follower ; hence in a single pair of wheels in gear together, one is the driver and the other the driven wheel or follower. But if there are three wheels in gear together, the middle one will be the follower when spoken of with reference to the first or prime mover, and the driver, when mentioned with reference to the third wheel, which will be a follower. A series of more than two wheels in gear together is termed a train of wheels or of gearing. When the wheels in a train are in gear continuously, so that each wheel, save the first and last, both receives and imparts motion, it is a simple train, the first wheel being the driver, and the last the follower, the others being termed intermediate wheels. Each of these intermediates is a follower with reference to the wheel that drives it, and a driver to the one that it drives. But the velocity of all the wheels in the train is the same in fact per second (or in a given space of time), although the revolutions in that space of time may vary; hence a simple train of wheels transmits motion without influencing its velocity. To alter the velocity (which is always taken at a point on the pitch circle) the gearing must be compounded, as in Fig. 11, in which A, B, C, E are four wheels in gear, B and C being compounded, that is, so held together on the shaft D that both make an equal number of revolutions in a given time. Hence the velocity of C will be less than that of B in proportion as the diameter, circumference, radius, or number of teeth in C, varies from the diameter, radius, circumference, or number of teeth (all the wheels being supposed to have teeth of the same pitch) in B, although the rotations of B and C are equal. It is most convenient, and therefore usual, to take the number of teeth, but if the teeth on C (and therefore those on E also) were of different pitch from those on B, the radius or diameters of the wheels must be taken instead of the pitch, when the velocities of the various wheels are to be computed. It is obvious that the compounded pair of wheels will diminish the velocity when the driver of the compounded pair (as C in the figure) is of less radius than the follower B, and conversely that the velocity will be increased when the driver is of greater radius than the follower of the compound pair.

The diameter of the addendum or outer circle of a wheel has no influence upon the velocity of the wheel. Suppose, for example, that we have a pair of wheels of 3 inch arc or circular

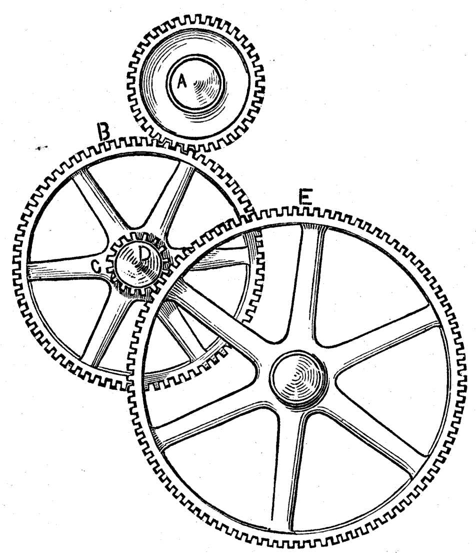

Fig. 11.

pitch, and containing 20 teeth, the driver of the two making one revolution per minute. Suppose the driven wheel to have fast upon its shaft a pulley whose diameter is one foot, and that a weight is suspended from a line or cord wound around this pulley, then (not taking the thickness of the line into account) each rotation of the driven wheel would raise the weight 3·1416 feet (that being the circumference of the pulley). Now suppose that the addendum circle of either of the wheels were cut off down to the pitch circle, and that they were again set in motion, then each rotation of the driven wheel would still raise the weight 3·1416 feet as before.

It is obvious, however, that the addendum circle must be sufficiently larger than the pitch circle to enable at least one pair of teeth to be in continuous contact; that is to say, it is obvious that contact between any two teeth must not cease before contact between the next two has taken place, for otherwise the motion would not be conveyed continuously. The diameter of the pitch circle cannot be obtained from that of the addendum circle unless the pitch of the teeth and the proportion of the pitch allowed for the addendum be known. But if these be known the diameter of the pitch circle may be obtained by subtracting from that of the addendum circle twice the amount allowed for the addendum of the tooth.

Example.—A wheel has 19 teeth of 3 inch arc pitch; the addendum of the tooth or teeth equals $\frac{3}{10}$ of the pitch, and its addendum circle measures 19·943 inches; what is the diameter of the pitch circle? Here the addendum on each side of the wheel equals ($\frac{3}{10}$ of 3 inches) = ·9 inches, hence the ·9 must be multiplied by 2 for the two sides of the wheel, thus, ·9 × 2 = 1·8. Then, diameter of addendum circle 19·943 inches less 1·8 inches = 18·143 inches, which is the diameter of the pitch circle.

Proof.—Number of teeth = 19, arc pitch 3, hence 19 × 3 = 57 inches, which, divided by 3·1416 (the proportion of the circumference to the diameter) = 18·143 inches.

If the distance between the centres of a pair of wheels that are in gear be divided into two parts whose lengths are in the same proportion one to the other as are the numbers of teeth in the wheels, then these two parts will represent the radius of the pitch circles of the respective wheels. Thus, suppose one wheel to contain 100 and the other 50 teeth, and that the distance between their centres is 18 inches, then the pitch radius or pitch diameter of one will be twice that of the other, because one contains twice as many teeth as the other. In this case the radius of pitch circle for the large wheel will be 12 inches, and that for the small one 6 inches, because 12 added to 6 makes 18, which is the distance between the wheel centres, and 12 is in the same proportion to 6 that 100 is to 50.

A simple rule whereby to find the radius of the pitch circles of a pair of wheels is as follows:—

Rule.—Divide number of teeth in the large wheel by the number in the small one, and to the sum so obtained add 1. Take this amount and divide it into the distance between the centres of the wheels, and the result will be the radius of the smallest wheel. To obtain the radius of the largest wheel subtract the radius of the smallest wheel from the distance between the wheel centres.

Example.—Of a pair of wheels, one has 100 and the other 50 teeth, the distance between their centres is 18 inches; what is the pitch radius of each wheel?

Here 100 ÷ 50 = 2, and 2 + 1 = 3. Then 18 ÷ 3 = 6, hence the pitch radius of the small wheel is 6 inches. Then 18 − 6 = 12 = pitch radius of large wheel.

Example 2.—Of a pair of wheels one has 40 and the other 90 teeth. The distance between the wheel centres is $32\frac{1}{2}$ inches; what are the radii of the respective pitch circles? 90 ÷ 40 = 2·25 and 2·25 + 1 = 3·25. Then 32·5 ÷ 3·25 = 10 = pitch radius of small wheel, and 32·5 − 10 = 22·5, which is the pitch radius of the large wheel.

To prove this we may show that the pitch radii of the two wheels are in the same proportion as their numbers of teeth, thus:—

$$\text{Proof.—}\frac{\text{Radius of small wheel}}{\text{radius of large wheel}} = \frac{10}{22\cdot 5} \times 4 = \frac{40}{90\cdot 0}$$

Suppose now that a pair of wheels are constructed, having respectively 50 and 100 teeth, and that the radii of their true pitch circles are 12 and 6 respectively, but that from wear in their journals or journal bearings this 18 inches (12 + 6 = 18) between centres (or line of centres, as it is termed) has become $18\frac{3}{8}$ inches. Then the acting effective or operative radii of the pitch circles will bear the same proportion to the $18\frac{3}{8}$ as the numbers of teeth in the respective wheels, and will be 12·25 for the large, and 6·125 for the small wheel, instead of 12 and 6, as would be the case were the wheels 18 inches apart. Working this out under the rule given we have 100 ÷ 50 = 2, and 2 + 1 = 3. Then 18·375 ÷ 3 = 6·125 = pitch radius of small wheel, and 18·375 − 6·125 = 12·25 = pitch radius of the large wheel.

The true pitch line of a tooth is the line or point where the face curve joins the flank curve, and it is essential to the transmission of uniform motion that the pitch circles of epicycloidal wheels exactly coincide on the line of centres, but if they do not coincide (as by not meeting or by overlapping each other), then a false pitch circle becomes operative instead of the true one, and the motion of the driven wheel will be unequal at different instants of time, although the revolutions of the wheels will of course be in proportion to the respective numbers of their teeth.

If the pitch circle is not marked on a single wheel and its arc pitch is not known, it is practically a difficult matter to obtain either the arc pitch or diameter of the pitch circle. If the wheel is a new one, and its teeth are of the proper curves, the pitch circle will be shown by the junction of the curves forming the faces with those forming the flanks of the teeth, because that is the location of the pitch circle ; but in worn wheels, where from play or looseness between the journals and their bearings, this point of junction becomes rounded, it cannot be defined with certainty.

In wheels of large diameter the arc pitch so nearly coincides with the chord pitch, that if the pitch circle is not marked on the wheel and the arc pitch is not known, the chord pitch is in practice often assumed to represent the arc pitch, and the diameter of the wheel is obtained by multiplying the number of teeth by the chord pitch. This induces no error in wheels of coarse pitches, because those pitches advance by $\frac{1}{4}$ or $\frac{1}{2}$ inch at a step, and a pitch measuring about, say, $1\frac{1}{4}$ inch chord pitch, would be known to be $1\frac{1}{4}$ arc pitch, because the difference between the arc and chord pitch would be too minute to cause sensible error. Thus the next coarsest pitch to 1 inch would be $1\frac{1}{8}$, or more often $1\frac{1}{4}$ inch, and the difference between the arc and chord pitch of the smallest wheel would not amount to anything near $\frac{1}{8}$ inch, hence there would be no liability to mistake a pitch of $1\frac{1}{8}$ for 1 inch or *vice versâ.* The diameter of wheel that will be large enough to transmit continuous motion is diminished in proportion as the pitch is decreased; in proportion, also, as the wheel diameter is reduced, the difference between the arc and chord pitch increases, and further the steps by which fine pitches advance are more minute (as $\frac{1}{4}$, $\frac{9}{32}$, $\frac{5}{16}$ &c.). From these facts there is much more liability to err in estimating the arc from the measured chord pitch in fine pitches, hence the employment of diametral pitch for small wheels of fine pitches is on this account also very advantageous. In marking out a wheel the chord pitch will be correct if the pitch circle be of correct diameter and be divided off into as many points of equal division (with compasses) as there are to be teeth in the wheel. We may then mark from these points others giving the thickness of the teeth, which will make the spaces also correct. But when the wheel teeth are to be cut in a machine out of solid metal, the mechanism of the machine enables the marking out to be dispensed with, and all that is necessary is to turn the wheel to the required addendum diameter, and mark the pitch circle. The following are rules for the purposes they indicate.

The circumference of a circle is obtained by multiplying its diameter by 3·1416, and the diameter may be obtained by dividing the circumference by 3·1416.

The circumference of the pitch circle divided by the arc pitch gives the number of teeth in the wheel.

The arc pitch multiplied by the number of teeth in the wheel gives the circumference of the pitch circle.

Gear-wheels are simply rotating levers transmitting the power they receive, less the amount of friction necessary to rotate them under the given conditions. All that is accomplished by a simple train of gearing is, as has been said, to vary the number of revolutions, the speed or velocity measured in feet moved through per minute remaining the same for every wheel in the train. But in a compound train of gears the speed in feet per minute, as well as the revolutions, may be varied by means of the compounded pairs of wheels. In either a simple or a compound train of gearing the power remains the same in amount for every wheel in the train, because what is in a compound train lost in velocity is gained in force, or what is gained in velocity is lost in force, the word force being used to convey the idea of strain, pressure, or pull.

In Fig. 12, let A, B, and C represent the pitch circles of three gears of which A and B are in gear, while C is compounded with B ; let E be the shaft of A, and G that for B and C. Let A be 60 inches, B=30 inches, and C=40 inches in diameter. Now suppose that shaft E suspends from its perimeter a weight of 50 lbs., the shaft being 4 inches in diameter. Then this weight will be at a leverage of 2 inches from the centre of E and the 50 must be multiplied by 2, making 100 lbs. at the centre of E. Then at the perimeter of A this 100 will become one-thirtieth of one hundred, because from the centre to the perimeter of A is 30. One-thirtieth of 100 is $3\frac{33}{100}$ lbs., which will be the force exerted by A on the perimeter of B. Now from the perimeter of B to its centre (or in other words its radius) is 15 inches, hence the $3\frac{33}{100}$ lbs. at its perimeter will become fifteen times as much at the centre G of B, and $3\frac{33}{100} \times 15 = 49\frac{95}{100}$ lbs. From the centre G to the perimeter of C being 20 inches, the $49\frac{95}{100}$ lbs. at the centre will be only one-twentieth of that amount at the perimeter of C, hence $49\frac{95}{100} \div 20 = 2\frac{49}{100}$ lbs., which is the amount of force at the perimeter of C.

Here we have treated the wheels as simple levers, dividing the weight by the length of the levers in all cases where it is transmitted from the shaft to the perimeter, and multiplying it by the length of the lever when it is transmitted from the perimeter of the wheel to the centre of the shaft. The precise same result will be reached if we take the diameter of the wheels or the number of the teeth, providing the pitch of the teeth on all the wheels is alike.

Suppose, for example, that A has 60 teeth, B has 30 teeth, and C has 40 teeth, all being of the same pitch. Suppose the 50 lb. weight be suspended as before, and that the circumference of the shaft be equal to that of a pinion having 4 teeth of the same pitch as the wheels. Then the 50 multiplied by the 4 becomes 200, which divided by 60 (the number of teeth on A) becomes $3\frac{33}{100}$, which multiplied by 30 (the number of teeth on B) becomes $99\frac{90}{100}$,

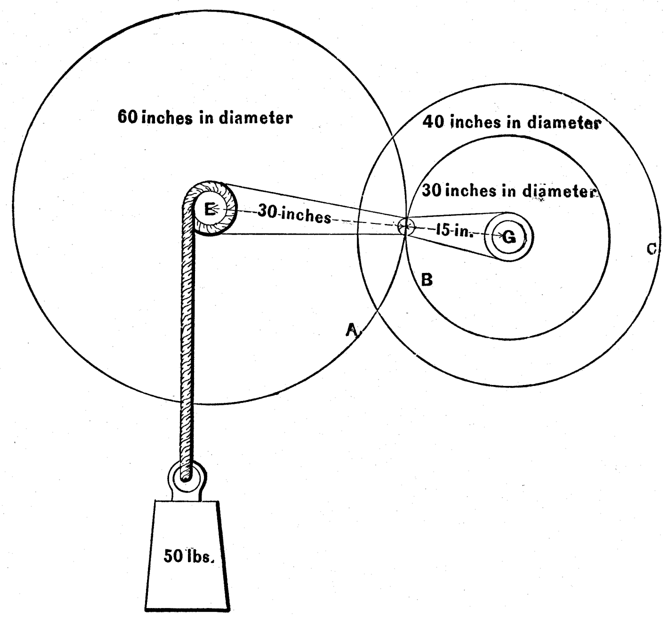

Fig. 12.

which divided by 40 (the number of teeth on C) becomes $2\frac{49}{100}$ lbs. as before.

It may now be explained why the shaft was taken as equal to a pinion having 4 teeth. Its diameter was taken as 4 inches and the wheel diameter was taken as being 60 inches, and it was supposed to contain 60 teeth, hence there was 1 tooth to each inch of diameter, and the 4 inches diameter of shaft was therefore equal to a pinion having 4 teeth. From this we may perceive the philosophy of the rule that to obtain the revolutions of wheels we multiply the given revolutions by the teeth in the driving wheels and divide by the teeth in the driven wheels.

Suppose that A (Fig. 13) makes 1 revolution per minute, how many will C make, A having 60 teeth, B 30 teeth, and C 40 teeth ? In this case we have but one drivingwheel A, and one driven wheel B, the driver having 60 teeth, the driven 30, hence $60 \div 30 = 2$, equals revolutions of B and also of C, the two latter being on the same shaft.

It will be observed then that the revolutions are in the same proportion as the numbers of the teeth or the radii of the wheels, or what is the same thing, in the same proportion as their diameters. The number of teeth, however, is usually taken as being easier obtained than the diameter of the pitch circles, and easier to calculate, because the teeth will be represented by a whole number, whereas the diameter, radius, or circumference, will generally contain fractions.

Suppose that the 4 wheels in Fig. 14 have the respective numbers of teeth marked beside them, and that the upper one having 40 teeth makes 60 revolutions per minute, then we may obtain the revolutions of the others as follows :—

Revolutions.		Teeth in first driver.		Teeth in first driven.		Teeth in second driver.		Teeth in second driven.		
60	×	40	÷	60	×	20	÷	120	=	$6\frac{66}{100}$

and a remainder of the reciprocating decimals. We may now prove this by reversing the question, thus. Suppose the 120 wheel to make $6\frac{66}{100}$ revolutions per minute, how many will the 40 wheel make ?

Revolutions.		Teeth in first driver.		Teeth in first driven.		Teeth in second driver.		Teeth in second driven.			
6·66	×	120	÷	20	×	60	÷	40	=	$59\frac{99}{100}$	=

revolutions of the 40 wheel, the discrepancy of $\frac{1}{100}$ being due to the 6·66 leaving a remainder and not therefore being absolutely correct.

That the amount of power transmitted by gearing, whether compounded or not, is equal throughout every wheel in the train, may be shown as follows :—

Referring again to Fig. 10, it has been shown that with a 50 lb.

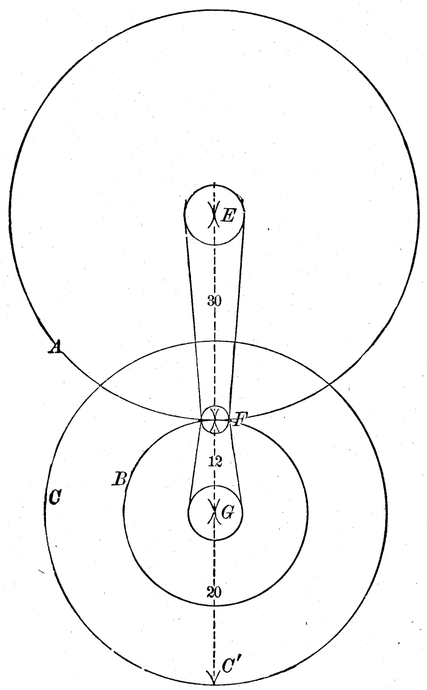

Fig. 13.

weight suspended from a 4 inch shaft E, there would be $30\frac{33}{100}$ lbs. at the perimeter of A. Now suppose a rotation be made, then the 50 lb. weight would fall a distance equal to the circumference of the shaft, which is (3.1416 × 4 = $12\frac{56}{100}$) $12\frac{56}{100}$ inches. Now the circumference of the wheel is (60 dia. × 3·1416 = $188\frac{49}{100}$ cir.) $188\frac{49}{100}$ inches, which is the distance through which the $3\frac{33}{100}$ lbs. would move during one rotation of A. Now 3·33 lbs. moving through 188·49 inches represents the same amount of power as does 50 lbs. moving through a distance of 12·56 inches, as may be found by converting the two into inch lbs. (that is to say, into the number of inches moved by 1 lb.), bearing in mind that there will be a slight discrepancy due to the fact that the fractions ·33 in the one case, and ·56 in the other are not quite correct. Thus :

188·49 inches × 3·33 lbs. = 627·67 inch lbs., and
12·56 ,, × 50 ,, = 628 ,, ,,

Taking the next wheels in Fig. 12, it has been shown that the 3·33 lbs. delivered from A to the perimeter of B, becomes 2·49 lbs. at the perimeter of C, and it has also been shown that C makes two revolutions to one of A, and its diameter being 40 inches, the distance this 2·49 lbs. will move through in one revolution of A will therefore be equal to twice its circumference, which is (40 dia. × 3·1416 = 125·666 cir., and 125·666 × 2 = 251·332) 251·332 inches. Now 2·49 lbs. moving through 251·332 gives when brought to inch lbs. 627.67 inch lbs., thus 251·332 × 2·49 = 627·67. Hence the amount of power remains constant, but is altered in form, merely being converted from a heavy weight moving a short distance, into a lighter one moving a distance exactly as much greater as the weight or force is lessened or lighter.

Gear-wheels therefore form a convenient method of either simply transmitting motion or power, as when the wheels are

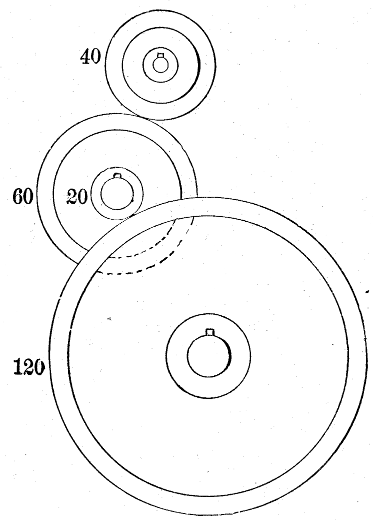

Fig. 14.

all of equal diameter, or of transmitting it and simultaneously varying its velocity of motion, as when the wheels are compounded either to reduce or increase the speed or velocity in feet per second of the prime mover or first driver of the train or pair, as the case may be.

In considering the action of gear-teeth, however, it sometimes is more convenient to denote their motion by the number of degrees of angle they move through during a certain portion of a revolution, and to refer to their relative velocities in terms of the ratio or proportion existing between their velocities. The first of

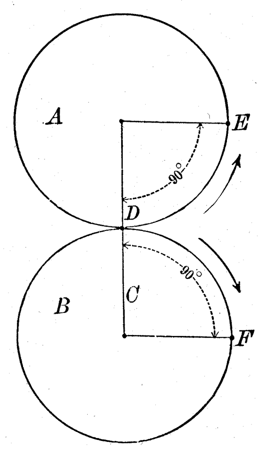

Fig. 15.

these is termed the angular velocity, or the number of degrees of angle the wheel moves through during a given period, while the second is termed the velocity ratio of the pair of wheels. Let it be supposed that two wheels of equal diameter have contact at their perimeters so that one drives the other by friction without any slip, then the velocity of a point on the perimeter of one will equal that of a point on the other. Thus in Fig. 15 let A and B represent the pitch circles of two wheels, and C an imaginary line joining the axes of the two wheels and termed the line of centres. Now the point of contact of the two wheels will be on the line of centres as at D, and if a point or dot be marked at D and motion be imparted from A to B, then when each wheel has made a quarter revolution the dot on A will have arrived at E while that on B will have arrived at F. As each wheel has moved through one quarter revolution, it has moved through 90° of angle, because in the whole circle there is 360°, one quarter of which is 90°, hence instead of saying that the wheels have each moved through one quarter of a revolution we may say they have moved through an angle of 90°, or, in other words, their angular velocity has, during this period, been 90°. And as both wheels have moved through an equal number of degrees of angle their velocity ratio or proportion of velocity has been equal.

Obviously then the angular velocity of a wheel represents a portion of a revolution irrespective of the diameter of the wheel, while the velocity ratio represents the diameter of one in proportion to that of the other irrespective of the actual diameter of either of them.

Now suppose that in Fig. 16 A is a wheel of twice the diameter of B; that the two are free to revolve about their fixed centres, but that there is frictional contact between their perimeters at the line of centres sufficient to cause the motion of one to be imparted to the other without slip or lost motion, and that a point be marked on both wheels at the point of contact D. Now let motion be communicated to A until the mark that was made at D has moved one-eighth of a revolution and it will have moved through an eighth of a circle, or 45°. But during this motion the mark on B will have moved a quarter of a revolution, or through an angle of

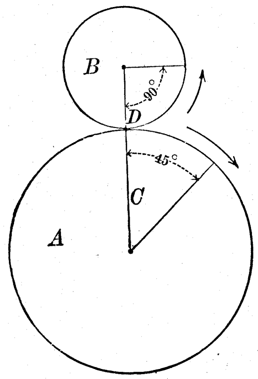

Fig. 16.

90° (which is one quarter of the 360° that there are in the whole circle). The angular velocities of the two are, therefore, in the same ratio as their diameters, or two to one, and the velocity ratio is also two to one. The angular velocity of each is therefore the number of degrees of angle that it moves through in a certain portion of a revolution, or during the period that the other wheel of the pair makes a certain portion of a revolution, while the velocity ratio is the proportion existing between the velocity of one wheel and that of the other; hence if the diameter of one only of the wheels be changed, its angular velocity will be changed and the velocity ratio of the pair will be changed. The velocity ratio may be obtained by dividing either the radius, pitch, diameter, or number of teeth of one wheel into that of the other.

Conversely, if a given velocity ratio is to be obtained, the radius, diameter, or number of teeth of the driver must bear the same relation to the radius, diameter, or number of teeth of the follower, as the velocity of the follower is desired to bear to that of the driver.

If a pair of wheels have an equal number of teeth, the same pairs of teeth will come into action at every revolution; but if of two wheels one is twice as large as the other, each tooth on the small wheel will come into action twice during each revolution of the large one, and will work during each successive revolution with the same two teeth on the large wheel; and an application of the principle of the hunting tooth is sometimes employed in clocks to prevent the overwinding of their springs, the device being shown in Fig. 17, which is from "Willis' Principles of Mechanism."

For this purpose the winding arbor C has a pinion A of 19 teeth fixed to it close to the front plate. A pinion B of 18 teeth is mounted on a stud so as to be in gear with the former. A radial plate C D is fixed to the face of the upper wheel A, and a similar plate F E to the lower wheel B. These plates terminate outward in semicircular noses D, E, so proportioned as to cause their extremities to abut against each other, as shown in the figure, when the motion given to the upper arbor by the winding has brought them into the position of contact. The clock being now wound up, the winding arbor and wheel A will begin to turn in the opposite direction. When its first complete rotation is effected the wheel B will have gained one tooth distance from the line of centres, so as to place the stop D in advance of E and thus avoid a contact with E, which would stop the motion. As each turn of the upper wheel increases the distance of the stops, it follows from the principle of the hunting cog, that after eighteen revolutions of A and nineteen of B the stops will come together again and the clock be prevented from running down too far. The winding key being applied, the upper wheel A will be rotated in the opposite direction, and the winding repeated as above.

Thus the teeth on one wheel will wear to imbed one upon the other. On the other hand the teeth of the two wheels may be of such numbers that those on one wheel will not fall into gear with the same teeth on the other except at intervals, and thus an inequality on any one tooth is subjected to correction by all the teeth in the other wheel. When a tooth is added to the

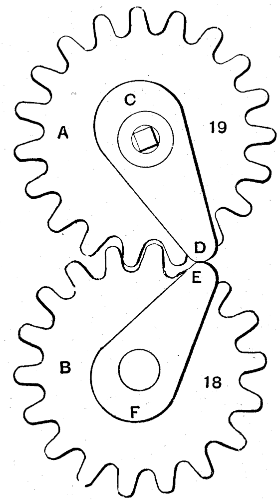

Fig. 17.

number of teeth on a wheel to effect this purpose it is termed a hunting cog, or hunting tooth, because if one wheel have a tooth less, then any two teeth which meet in the first revolution are distant, one tooth in the second, two teeth in the third, three in the fourth, and so on. The odd tooth is on this account termed a hunting tooth.

It is obvious then that the shape or form to be given to the teeth must, to obtain correct results, be such that the motion of the driver will be communicated to the follower with the velocity due to the relative diameters of the wheels at the pitch circles, and since the teeth move in the arc of a circle it is also obvious that the sides of the teeth, which are the only parts that come into contact, must be of same curve. The nature of this curve must be such that the teeth shall possess the strength necessary to transmit the required amount of power, shall possess ample wearing surface, shall be as easily produced as possible for all the varying conditions, shall give as many teeth in constant contact as possible, and shall, as far as possible, exert a pressure in a direction to rotate the wheels without inducing under wear upon the journals of the shafts upon which the wheels rotate. In cases, however, in which some of these requirements must be partly sacrificed to increase the value of the others, or of some of the others, to suit the special circumstances under which the wheels are to operate, the selection is left to the judgment of the designer, and the considerations which should influence his determinations will appear hereafter.

Modern practice has accepted the curve known in general terms as the cycloid, as that best filling all the requirements of wheel teeth, and this curve is employed to produce two distinct forms of teeth, epicycloidal and involute. In epicycloidal teeth the curve forming the face of the tooth is designated an epicycloid, and that forming the flank an hypocycloid. An epicycloid may be traced or generated, as it is termed, by a point in the circumference of a circle that rolls without slip upon the circumference of another circle. Thus, in Fig. 18, A and B represent two wooden wheels, A having a pencil at P, to serve as a tracing or marking point. Now, if the wheels are laid upon a sheet of

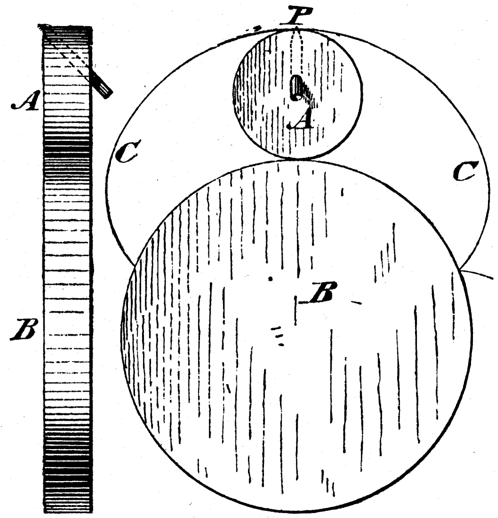

Fig. 18.

paper and while holding B in a fixed position, roll A in contact with B and let the tracing point touch the paper, the point P will trace the curve C C. Suppose now the diameter of the base circle B to be infinitely large, a portion of its circumference may be represented by a straight line, and the curve traced by a point on the circumference of the generating circle as it rolls along the base line B is termed a cycloid. Thus, in Fig. 19, B is the base line, A the rolling wheel or generating circle, and C C the cycloidal curve traced or marked by the point D when A is rolled along B. If now we suppose the base line B to represent the pitch line of a rack, it will be obvious that part of the cycloid at

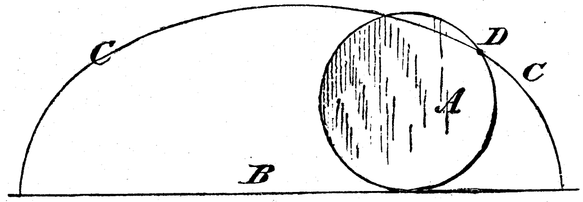

Fig. 19.

one end is suitable for the face on one side of the tooth, and a part at the other end is suitable for the face of the other side of the tooth.

A hypocycloid is a curve traced or generated by a point on the circumference of a circle rolling within and in contact (without slip) with another circle. Thus, in Fig. 20, A represents a wheel in contact with the internal circumference of B, and a point on its circumference will trace the two curves, C C, both curves starting from the same point, the upper having been traced by rolling the generating circle or wheel A in one direction and the lower curve by rolling it in the opposite direction.

To demonstrate that by the epicycloidal and hypocycloidal curves, forming the faces and flanks of what are known as epicycloidal teeth, motion may be communicated from one wheel to another with as much uniformity as by frictional contact of their circumferential surfaces, let A, B, in Fig. 21, represent two plain wheel disks at liberty to revolve about their fixed centres, and let C C represent a margin of stiff white paper attached to the face of B so as to revolve with it. Now suppose that A and B are in close contact at their perimeters at the point G, and that there is no slip, and that rotary motion commenced when the point E (where as tracing point a pencil is attached), in conjunction with the point F, formed the point of contact of the two wheels, and

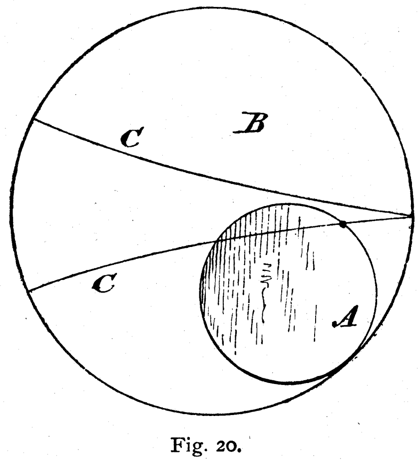

Fig. 20.

continued until the points E and F had arrived at their respective positions as shown in the figure; the pencil at E will have traced upon the margin of white paper the portion of an epicycloid denoted by the curve E F; and as the movement of the two wheels A, B, took place by reason of the contact of their circumferences, it is evident that the length of the arc E G must be equal to that

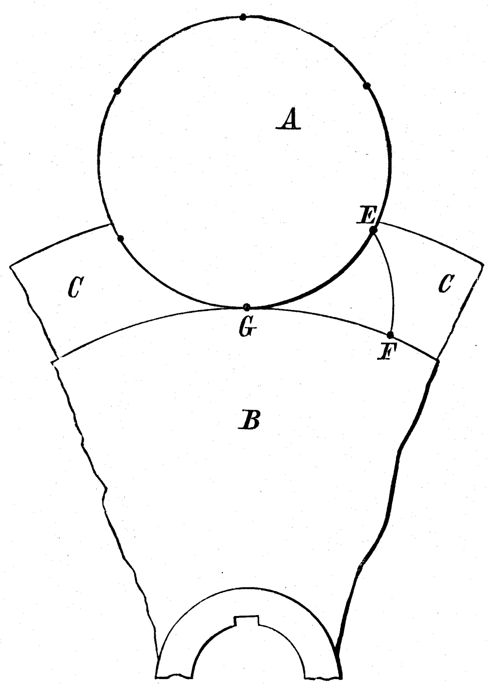

Fig. 21.

of the arc G F, and that the motion of A (supposing it to be the driver) would be communicated uniformly to B.

Now suppose that the wheels had been rotated in the opposite direction and the same form of curve would be produced, but it would run in the opposite direction, and these two curves may be utilized to form teeth, as in Fig. 22, the points on the wheel A working against the curved sides of the teeth on B.

To render such a pair of wheels useful in practice, all that is necessary is to diminish the teeth on B without altering the nature of the curves, and increase the diameter of the points on A, making them into rungs or pins, thus forming the wheels into what is termed a wheel and lantern, which are illustrated in Fig. 23.

A represents the pinion (or lantern), and B the wheel, and C, C, the primitive teeth reduced in thickness to receive the pins on

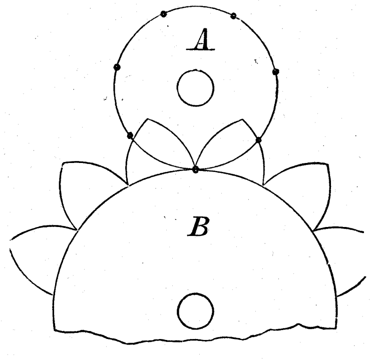

Fig. 22.

A. This reduction we may make by setting a pair of compasses to the radius of the rung and describing half-circles at the bottom of the spaces in B. We may then set a pair of compasses to the curve of C, and mark off the faces of the teeth of B to meet the

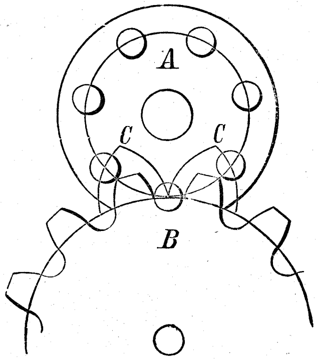

Fig. 23.

half-circles at the pitch line, and reduce the teeth heights so as to leave the points of the proper thickness; having in this operation maintained the same epicycloidal curves, but brought them closer together and made them shorter. It is obvious, however,

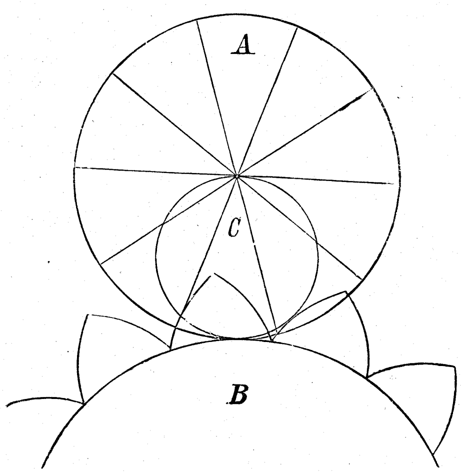

Fig. 24.

that such a method of communicating rotary motion is unsuited to the transmission of much power; because of the weakness of, and small amount of wearing surface on, the points or rungs in A.

In place of points or rungs we may have radial lines, these lines, representing the surfaces of ribs, set equidistant on the radial face of the pinion, as in Fig. 24. To determine the epicycloidal curves for the faces of teeth to work with these radial lines, we may take a generating circle C, of half the diameter of A, and cause it to roll in contact with the internal circumference of A, and a tracing point fixed in the circumference of C will

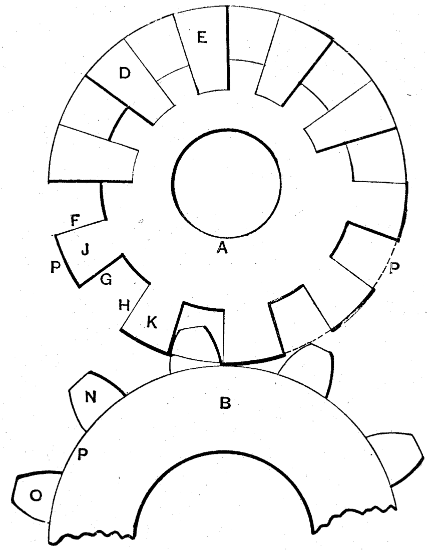

Fig. 25.

draw the radial lines shown upon A. The circumstances will not be altered if we suppose the three circles, A, B, C, to be movable about their fixed centres, and let their centres be in a straight line; and if, under these circumstances, we suppose rotation to be imparted to the three circles, through frictional contact of their perimeters, a tracing point on the circumference of C would trace the epicycloids shown upon B and the radial lines shown upon A, evidencing the capability of one to impart uniform rotary motion to the other.

To render the radial lines capable of use we must let them be the surfaces of lugs or projections on the face of the wheel, as shown in Fig. 25 at D, E, &c., or the faces of notches cut in the wheel as at F, G, H, &c., the metal between F and G forming a tooth J, having flanks only. The wheel B has the curves of each

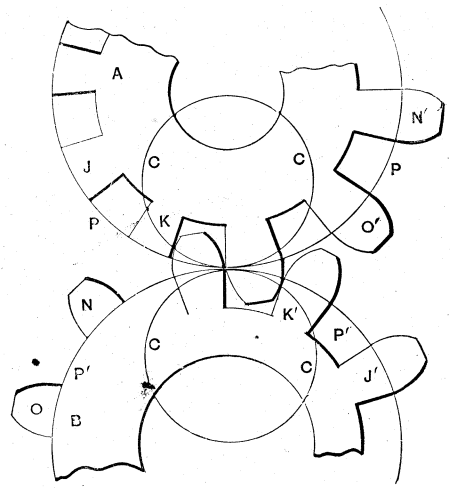

Fig. 26.

tooth brought closer together to give room for the reception of the teeth upon A. We have here a pair of gears that possess sufficient strength and are capable of working correctly in either direction.

But the form of tooth on one wheel is conformed simply to suit those on the other, hence, neither two of the wheels A, nor would two of B, work correctly together.

They may be qualified to do so, however, by simply adding to the tops of the teeth on A, teeth of the form of those on B, and adding to those on B, and within the pitch circle, teeth corresponding to those on A, as in Fig. 26, where at K′ and J′ teeth are provided on B corresponding to J and K on A, while on A there are added teeth O′, N′, corresponding to O, N, on B, with the result that two wheels such as A or two such as B would work correctly together, either being the driver or either the follower, and rotation may occur in either direction. In this operation we have simply added faces to the teeth on A, and flanks to those on B, the curves being generated or obtained by rolling the generating, or curve marking, circle C upon the pitch circles P and P′. Thus, for the flanks of the teeth of A, C is rolled upon, and within the pitch circle P of A; while for the face curves of the same teeth C is rolled upon, but without or outside of P. Similarly for the teeth of wheel B the generating circle C is rolled within P′ for the flanks and without for the faces. With the curves rolled or produced with the same diameter of generating circle the wheels will work correctly together, no matter what their relative diameter may be, as will be shown hereafter.

to wheel Q a pencil whose point is at *n*. If then rotation be given to *a a* in the direction of the arrow *s*, all three wheels will rotate in that direction as denoted by their respective arrows *s*.

Assume, then, that rotation of the three has occurred until the pencil point at *n* has arrived at the point *m*, and during this period of rotation the point *n* will recede from the line of centres A B, and will also recede from the arcs or lines of the two pitch circles *a a*, *b b*. The pencil point being capable of marking its path, it will be found on reaching *m* to have marked inside the pitch circle *b b* the curve denoted by the full line *m x*, and simultaneously with this curve it has marked another curve outside of *a a*, as denoted by the dotted line *y m*. These two curves being marked by the pencil point at the same time and extending from *y* to *m*, and *x* also to *m*. They are prolonged respectively to *p* and to K for clearness of illustration only.

The rotation of the three wheels being continued, when the pencil point has arrived at O it will have continued the same curves as shown at O *f*, and O *g*, curve O *f* being the same as

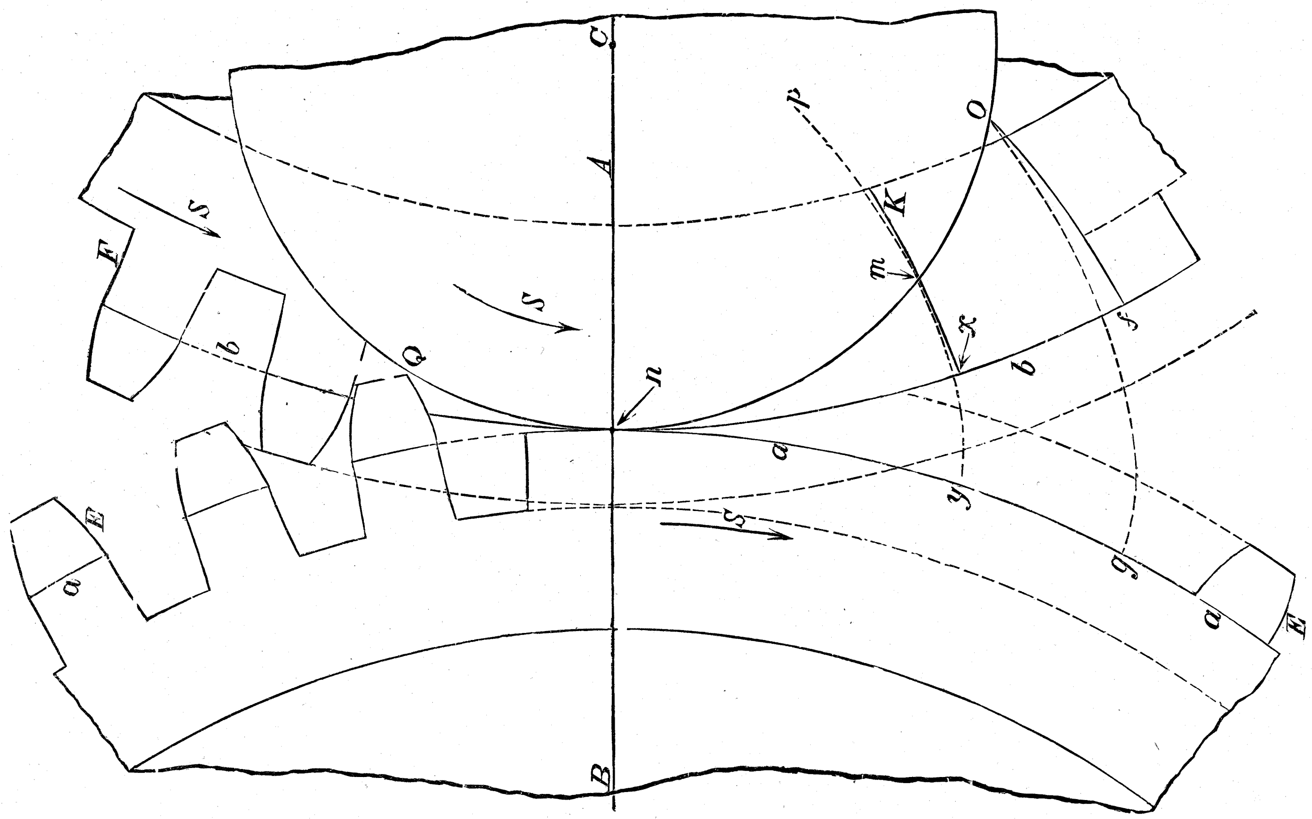

Fig. 27.

In this demonstration, however, the curves for the faces of the teeth being produced by an operation distinct from that employed to produce the flank curves, it is not clearly seen that the curves for the flanks of one wheel are the proper curves to insure a uniform velocity to the other. This, however, may be made clear as follows :—

In Fig. 27 let *a a* and *b b* represent the pitch circles of two wheels of equal diameters, and therefore having the same number of teeth. On the left, the wheels are shown with the teeth in, while on the right-hand side of the line of centres A B, the wheels are shown blank ; *a a* is the pitch line of one wheel, and *b b* that for the other. Now suppose that both wheels are capable of being rotated on their shafts, whose centres will of course be on the line A B, and suppose a third disk, Q, be also capable of rotation upon its centre, C, which is also on the line A B. Let these three wheels have sufficient contact at their perimeters at the point *n*, that if one be rotated it will rotate both the others (by friction) without any slip or lost motion, and of course all three will rotate at an equal velocity. Suppose that there is fixed

m x placed in a new position, and O *g* being the same as *m y*, but placed in a new position. Now since both these curves (O *f* and O *g*) were marked by the one pencil point, and at the same time, it follows that at every point in its course that point must have touched both curves at once. Now the pencil point having moved around the arc of the circle Q from *n* to *m*, it is obvious that the two curves must always be in contact, or coincide with each other, at some point in the path of the pencil or describing point, or, in other words, the curves will always touch each other at some point on the curve of Q, and between *n* and O. Thus when the pencil has arrived at *m*, curve *m y* touches curve K *x* at the point *m*, while when the pencil had arrived at point O, the curves O *f* and O *g* will touch at O. Now the pitch circles *a a* and *b b*, and the describing circle Q, having had constant and uniform velocity while the traced curves had constant contact at some point in their lengths, it is evident that if instead of being mere lines, *m y* was the face of a tooth on *a a*, and *m x* was the flank of a tooth on *b b*, the same uniform motion may be transmitted from *a a*, to *b b*, by pressing the tooth face *m y* against the tooth flank *m x*. Let it now be noted that the curve *y m* corresponds to the face of a tooth, as say the face E of a tooth on *a a*, and that curve *x m* corresponds to the flank of a tooth on *b b*, as say to the flank F, short portions only of the curves being used for those flanks. If the direction of rotation of the three wheels was reversed, the same shape of curves would be produced, but they would lie in an opposite direction, and would, therefore, be suitable for the other sides of the teeth. In this case, the contact of tooth upon tooth will be on the other side of the line of centres, as at some point between *n* and Q.

In this illustration the diameter of the rolling or describing circle Q, being less than the radius of the wheels *a a* or *b b*, the flanks of the teeth are curves, and the two wheels being of the same diameter, the teeth on the two are of the same shape But the principles governing the proper formation of the curve remain the same whatever be the conditions. Thus in Fig. 28 are segments of a pair of wheels of equal diameter, but the describing, rolling, or curve-generating circle is equal in diameter

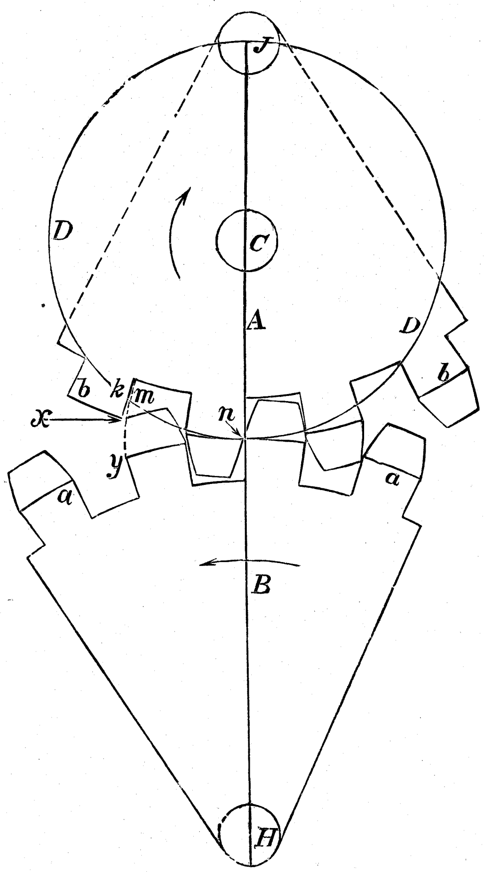

Fig. 28.

to the radius of the wheels. Motion is supposed to have occurred in the direction of the arrows, and the tracing point to have moved from *n* to *m*. During this motion it will have marked a curve *y m*, a portion of the *y* end serving for the face of a tooth on one wheel, and also the line *k x*, a continuation of which serves for the flank of a tooth on the other wheel. In Fig. 29 the pitch circles only of the wheels are marked, *a a* being twice the diameter of *b b*, and the curve-generating circle being equal in diameter to the radius of wheel *b b*. Motion is assumed to have occurred until the pencil point, starting from *n*, had arrived at *o*, marking curves suitable for the face of the teeth on one wheel and for the flanks of the other as before, and the contact of tooth upon tooth still, at every point in the path of the teeth, occurring at some point of the arc *n o*. Thus when the point had proceeded as far as point *m* it will have marked the curve *y* and the radial line *x*, and when the point had arrived at *o*, it will have prolonged *m y* into *o g* and *x* into *o f*, while in either position the point is marking both lines. The velocities of the wheels remain the same notwithstanding their different diameters, for the arc *n g* must obviously (if the wheels rotate without slip by friction of their surfaces while the curves are traced) be equal in length to the arc *n f* or the arc *n o*.

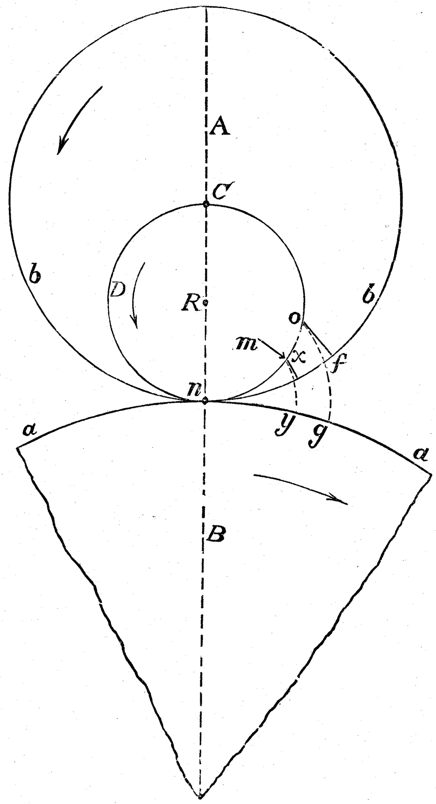

Fig. 29.

In Fig. 30 *a a* and *b b* are the pitch circles of two wheels as before, and *c c* the pitch circle of an annular or internal gear, and D is the rolling or describing circle. When the describing

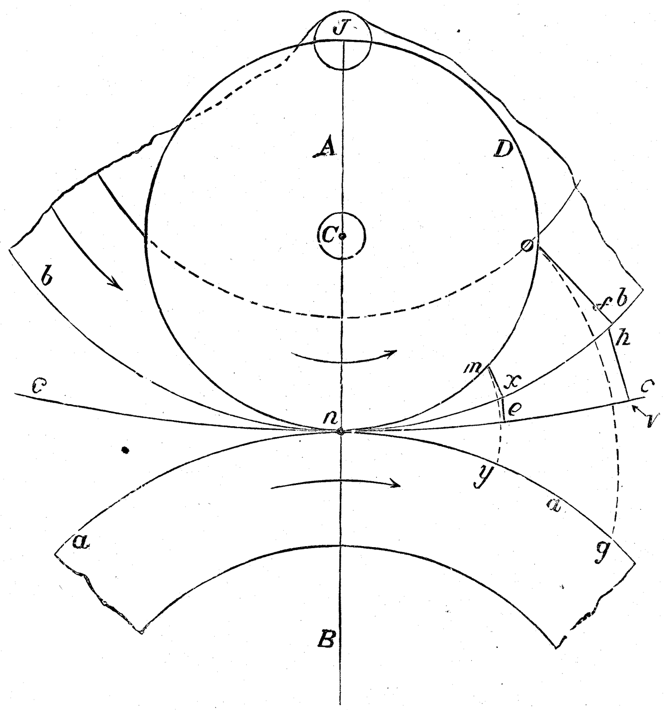

Fig. 30.

point arrived at *m*, it will have marked the curve *y* for the face of a tooth on *a a*, the curve *x* for the flank of a tooth on *b b*, and the curve *e* for the face of a tooth on the internal wheel *c c*.

Motion being continued *m y* will be prolonged to *o g*, while simultaneously *x* will be extended into *o f* and *e* into *h v*, the velocity of all the wheels being uniform and equal. Thus the arcs *n v*, *n f*, and *n g*, are of equal length.

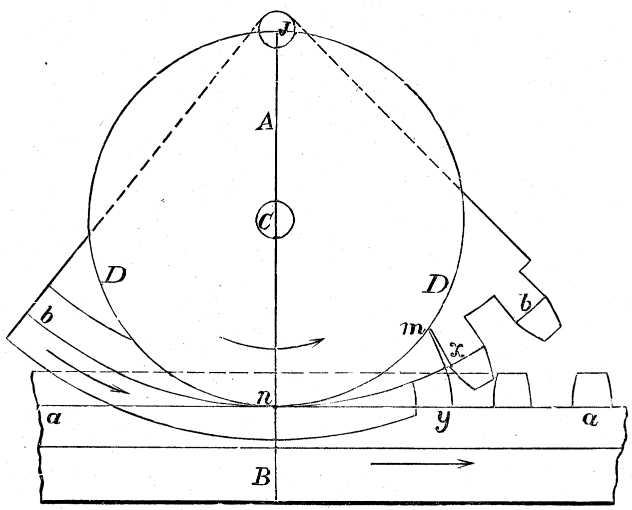

Fig. 31.

In Fig. 31 is shown the case of a rack and pinion; *a a* is the pitch line of the rack, *b b* that of the pinion, A B at a right angle to *a a*, the line of centres, and D the generating circle. The wheel and rack are shown with teeth *n* on one side simply for rolled around *a a* until it had reached the position marked 1, then it will have marked the curve from *e* to *n*, a part of this curve serving for the face of tooth *c*. Now let the rolling circle be placed within the pitch circle *a a* and its pencil point *n* be set to *e*, then, on being rolled to position 2, it will have marked the flank of tooth *c*. For the other wheel suppose the rolling wheel or circle to have started from *f* and rolled to the line of centres as in the cut, it will have traced the curve forming the face of the tooth *d*. For the flank of *d* the rolling circle or wheel is placed within *b b*, its tracing point set at *f* on the pitch circle, and on being rolled to position 3 it will have marked the flank curve. The curves thus produced will be precisely the same as those produced by rotating all three wheels about their axes, as in our previous demonstrations.

The curves both for the faces and for the flanks thus obtained will vary in their curvature with every variation in either the diameter of the generating circle or of the base or pitch circle of the wheel. Thus it will be observable to the eye that the face curve of tooth *c* is more curved than that of *d*, and also that the flank curve of *d* is more spread at the root than is that for *c*, which has in this case resulted from the difference between the diameter of the wheels *a a* and *b b*. But the curves obtained by a given diameter of rolling circle on a given diameter of pitch circle will be correct for any pitch of teeth that can be used upon wheels having that diameter of pitch circle. Thus, suppose we have a curve obtained by rolling a wheel of 20 inches circumference on a pitch circle of 40 inches circumference—now a wheel of 40 inches in circumference may contain 20 teeth of 2 inch arc pitch, or 10 teeth of 4 inch arc pitch, or 8 teeth of 5 inch arc pitch, and the curve may be used for either of those pitches.

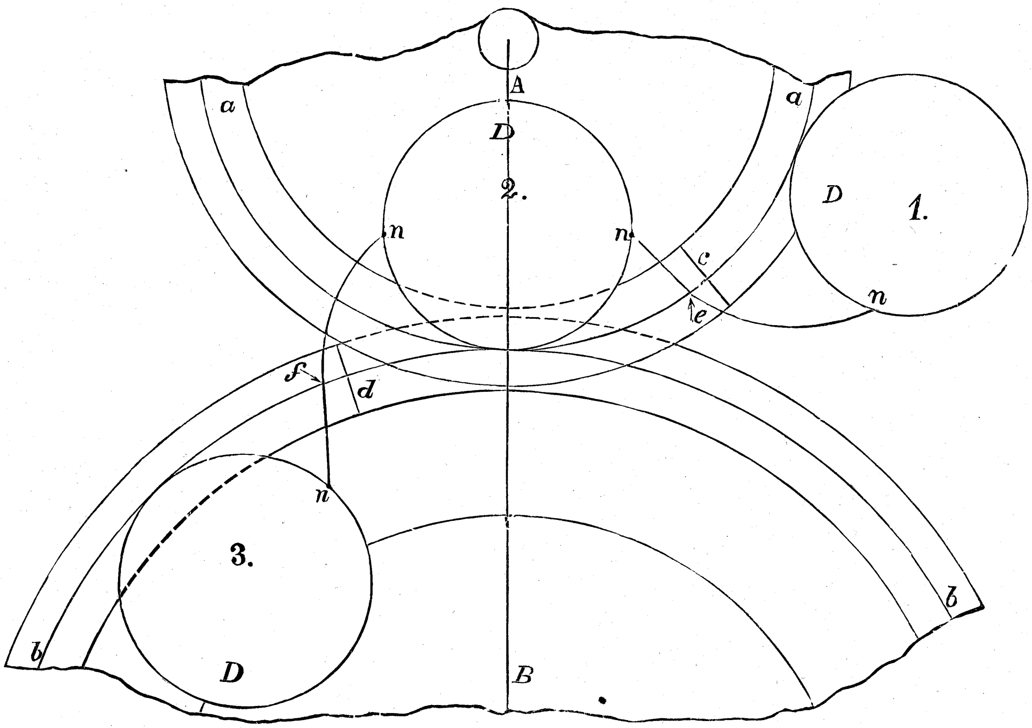

Fig. 32.

clearness of illustration. The pencil point *n* will, on arriving at *m*, have traced the flank curve *x* and the curve *y* for the face of the rack teeth.

It has been supposed that the three circles rotated together by the frictional contact of their perimeters on the line of centres, but the circumstances will remain the same if the wheels remain at rest while the generating or describing circle is rolled around them. Thus in Fig. 32 are two segments of wheels as before, *c* representing the centre of a tooth on *a a*, and *d* representing the centre of a tooth on *b b*. Now suppose that a generating or rolling circle be placed with its pencil point at *e*, and that it then be

If we trace the path of contact of each tooth, from the moment it takes until it leaves contact with a tooth upon the other wheel, we shall find that contact begins at the point where the flank of the tooth on the wheel that drives or imparts motion to the other wheel, meets the face of the tooth on the driven wheel, which will always be where the point of the driven tooth cuts or meets the generating or rolling circle of the driving tooth. Thus in Fig. 33 are represented segments of two spur-wheels marked respectively the driver and the driven, their generating circles being marked at G and G′, and X X representing the line of centres. Tooth A is shown in the position in which it commences its contact with tooth

B at C. Secondly, we shall find that as these two teeth approach the line of centres X, the point of contact between them moves or takes place along the thickened arc or curve C X, or along the path of the generating circle G.

Thus we may suppose tooth D to be another position of tooth A, the contact being at F, and as motion was continued the contact would pass along the thickened curve until it arrived at the line of centres X. Now since the teeth have during this path of contact approached the line of centres, this part of the whole arc of action or of the path of contact is termed the arc of approach. After the two teeth have passed the line of centres X, the path of contact of the teeth will be along the dotted arc from X to L, and as the teeth are during this period of motion receding from X this part of the contact path is termed the arc of recess.

That contact of the teeth would not occur earlier than at C nor later than at L, is shown by the dotted teeth sides; thus A and B would not touch when in the position denoted by the dotted teeth, nor would teeth I and K if in the position denoted by their dotted lines.

If we examine further into this path of contact we find that throughout its whole path the face of the tooth of one wheel has contact with the flank only of the tooth of the other wheel, and also that the flank only of the driving-wheel tooth has contact before the tooth reaches the line of centres, while the face of only the driving tooth has contact after the tooth has passed the line of centres.

Thus the flanks of tooth A and of tooth D are in driving contact with the faces of teeth B and E, while the face of tooth H is in contact with the flank of tooth I.

These conditions will always exist, whatever be the diameters of the wheels, their number of teeth or the diameter of the generating circle. That is to say, in fully developed epicycloidal teeth, no matter which of two wheels is the driver or which the driven wheel, contact on the teeth of the driver will always be on the tooth flank during the arc of approach and on the tooth face during the arc of recess; while on the driven wheel contact during the arc of approach will be on the tooth face only, and during the arc of recess on the tooth flank only, it being borne in mind that the arcs of approach and recess are reversed in location if the direction of revolution be reversed. Thus if the direction of wheel motion was opposite to that denoted by the arrows in Fig. 33 then the arc of approach would be from M to X, and the arc of recess from X to N.

It is laid down by Professor Willis that the motion of a pair of gear-wheels is smoother in cases where the path of contact begins at the line of centres, or, in other words, when there is no arc of approach; and this action may be secured by giving to the driven wheel flanks only, as in Fig. 34, in which the driver has fully developed teeth, while the teeth on the driven have no faces.

In this case, supposing the wheels to revolve in the direction of arrow P, the contact will begin at the line of centres X, move or pass along the thickened arc and end at B, and there will be contact during the arc of recess only. Similarly, if the direction of motion be reversed as denoted by arrow Q, the driver will begin contact at X, and cease contact at H, having, as before, contact during the arc of recess only.

But if the wheel W were the driver and V the driven, then these conditions would be exactly reversed. Thus, suppose this to be the case and the direction of motion be as denoted by arrow P, the contact would occur during the arc of approach, from H to X, ceasing at X.

Or if W were the driver, and the direction of motion was as denoted by Q, then, again, the path of contact would be during the arc of approach only, beginning at B and ceasing at X, as denoted by the thickened arc B X.

The action of the teeth will in either case serve to give a theoretically perfect motion so far as uniformity of velocity is concerned, or, in other words, the motion of the driver will be transmitted with perfect uniformity to the driven wheel. It will be observed, however, that by the removal of the faces of the teeth, there are a less number of teeth in contact at each instant of time; thus, in Fig. 33 there is driving contact at three points, C, F, and J, while in Fig. 34 there is driving contact at two points only. From the fact that the faces of the teeth work with the flanks only, and that one side only of the teeth comes into action, it becomes apparent that each tooth may have curves formed by four different diameters of rolling or generating circles and yet work correctly, no matter which wheel be the driver, or which the driven wheel or follower, or in which direction motion occurs. Thus in Fig. 35, suppose wheel V to be the driver, having motion in the direction of arrow P, then faces A on the teeth of V will work with flanks B of the teeth on W, and so long as the curves for these faces and flanks are obtained with the same diameter of rolling circle, the action of the teeth will be correct, no matter

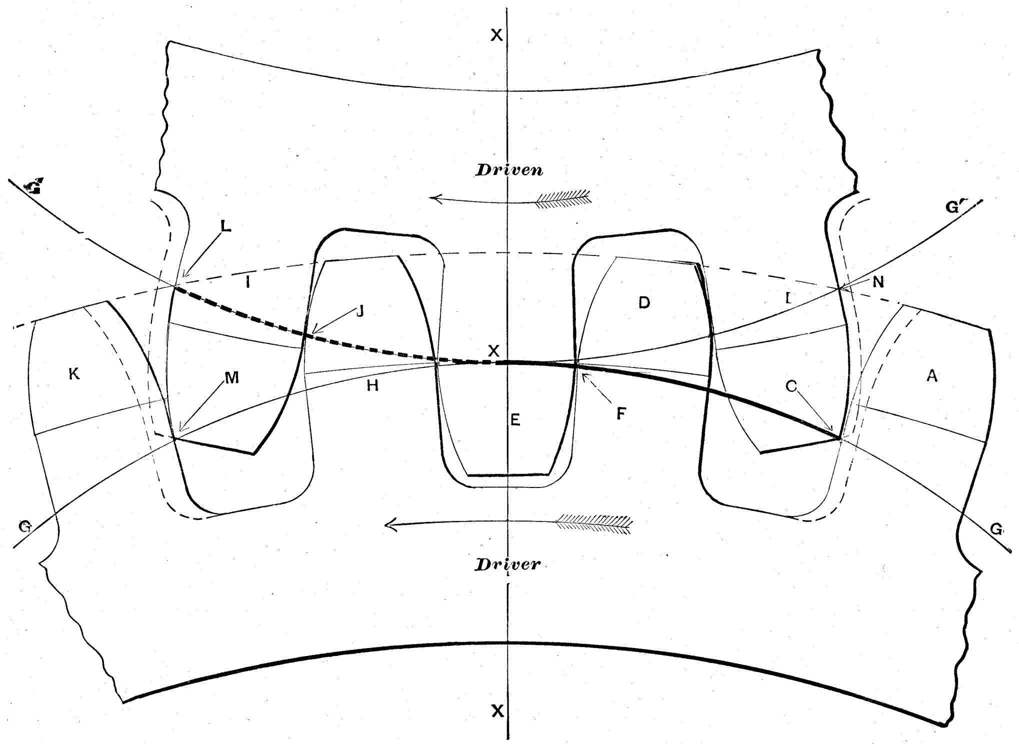

Fig. 33.

what the shapes of the other parts of the teeth. Now suppose that V still being the driver, motion occurs in the other direction as denoted by Q, then the faces C of the teeth on V will drive the flanks D of the teeth on W, and the motion will again be correct, providing that the same diameter (whatever it may be) of rolling circle be used for these faces and flanks, irrespective, of course, of what diameter of rolling circle is used for any other of the teeth curves. Now suppose that W is the driver, motion occurring in the direction of P, then faces E will drive flanks F, and the motion will be correct as before if the curves E and F are produced with the same diameter of rolling circle. Finally, let W be the driving wheel and motion occur in the direction of Q, and faces G will drive flanks H, and yet another diameter of rolling circle may be used for these faces and flanks. Here then it is shown that four different diameters of rolling circles may be used upon a pair of wheels, giving teeth-forms that will fill all the requirements so far as correctly transmitting motion is concerned. In the case of a pair of wheels having an equal number of teeth, so that each tooth on one wheel will always fall into gear with the same tooth on the other wheel, every tooth may have its individual curves differing from all the others, providing that the corresponding teeth on the other wheel are formed to match them by using the same size of rolling circle for each flank and face that work together.

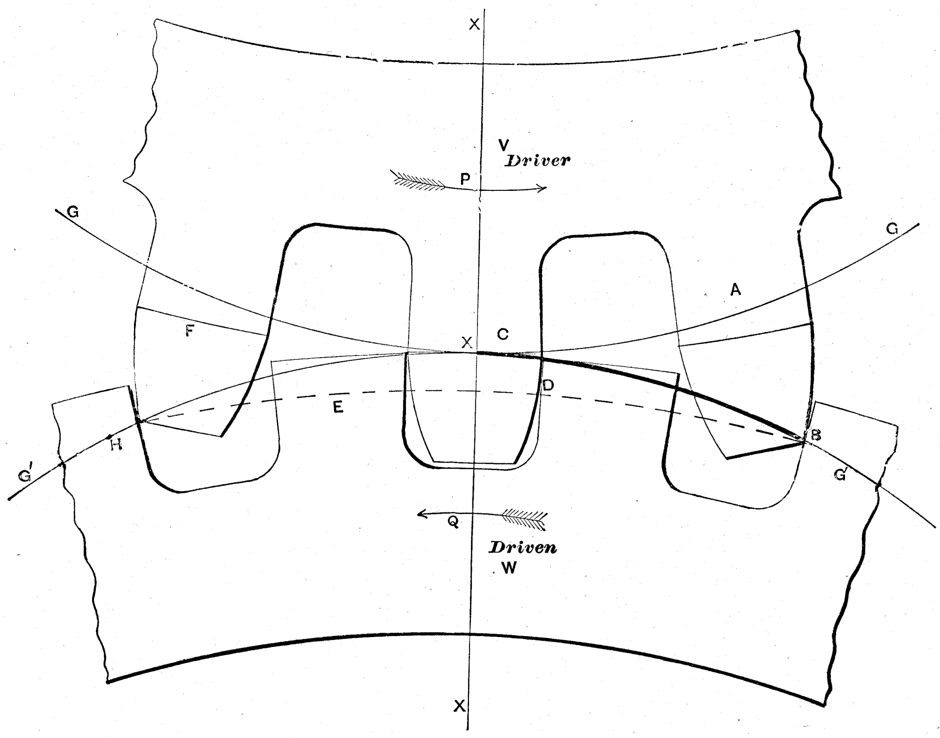

Fig. 34.

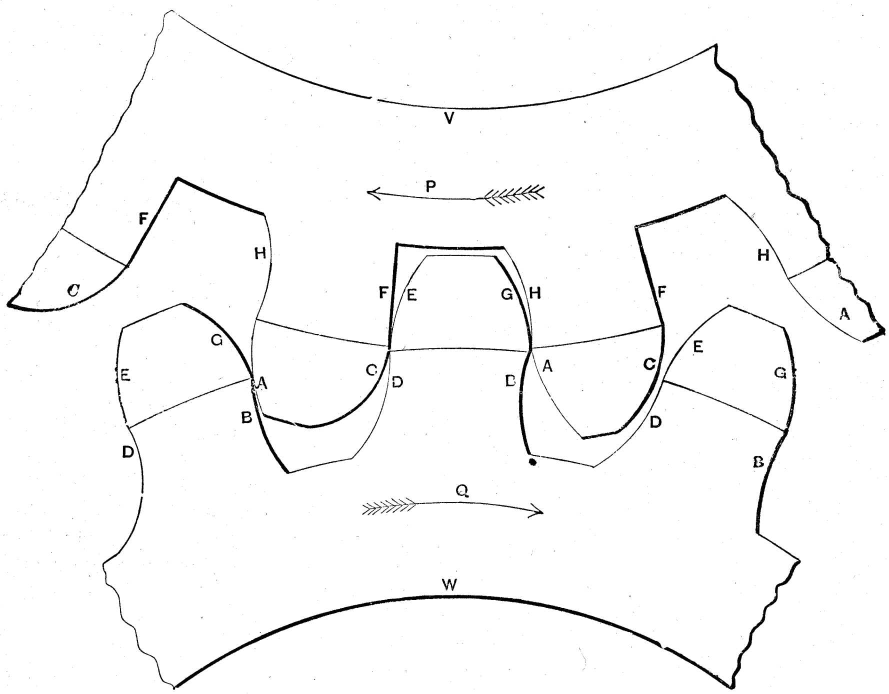

Fig. 35.

It is obvious, however, that such teeth would involve a great deal of labor in their formation and would possess no advantage, hence they are not employed. It is not unusual, however, in a pair of wheels that are to gear together and that are not intended to interchange with other wheels, to use such sizes as will give to both wheels teeth having radial flanks; which is done by using for the face of the teeth on the largest wheel of the pair and for the flanks of the teeth of the smallest wheel, a generating circle equal in diameter to the radius of the smallest wheel, and for the faces of the teeth of the small wheel and the flanks of the teeth of the large one, a generating circle whose diameter equals the radius of the large wheel.

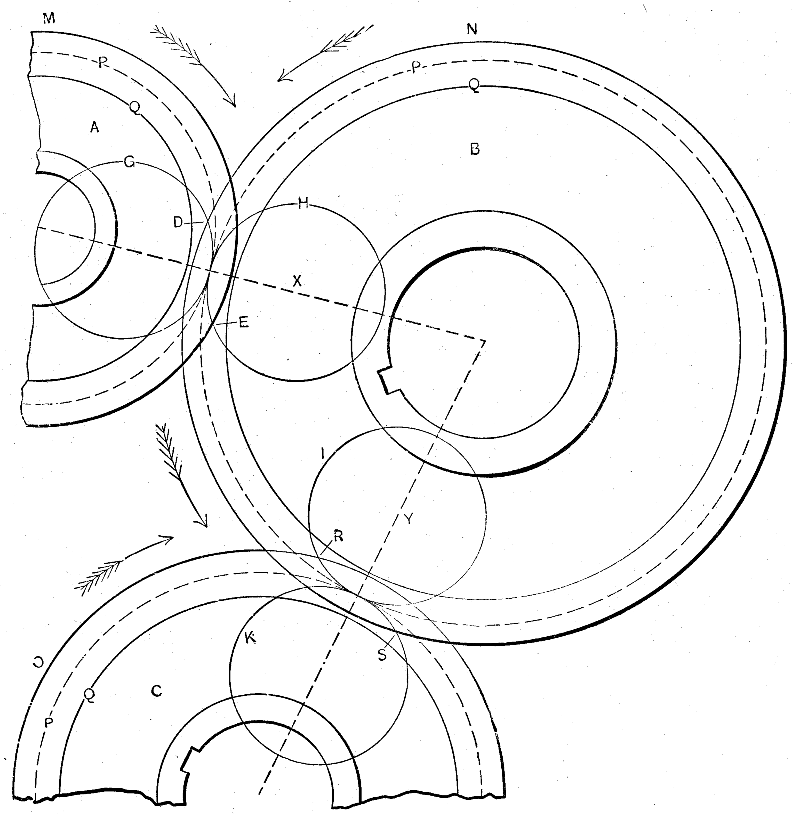

Fig. 36.

It will now be evident that if we have planned a pair or a train of wheels we may find how many teeth will be in contact for any given pitch, as follows. In Fig. 36 let A, B, and C, represent three blanks for gear-wheels whose addendum circles are M, N and O; P representing the pitch circles, and Q representing the circles for the roots of the teeth. Let X and Y represent the lines of centres, and G, H, I and K the generating or rolling circle, whose centres are on the respective lines of centres—the diameter of the generating circle being equal to the radius of the pinion, as in the Willis system, then, the pinion M being the driver, and the wheels revolving in the direction denoted by the respective arrows, the arc or path of contact for the first pair will be from point D, where the generating circle G crosses circle N to E, where generating circle H crosses the circle M, this path being composed of two arcs of a circle. All that is necessary, therefore, is to set the compasses to the pitch the teeth are to have and step them along these arcs, and the number of steps will be the number of teeth that will be in contact. Similarly, for the second pair contact will begin at R and end at S, and the compasses applied as before (from R to S) along the arc of generating circle I to the line of centres, and thence along the arc of generating circle K to S, will give in the number of steps, the number of teeth that will be in contact. If for any given purpose the number of teeth thus found to be in contact is insufficient, the pitch may be made finer.

When a wheel is intended to be formed to work correctly with any other wheel having the same pitch, or when there are more than two wheels in the train, it is necessary that the same size of generating circle be used for all the faces and all the flanks in the set, and if this be done the wheels will work correctly together, no matter what the number of the teeth in each wheel may be, nor in what way they are interchanged. Thus in Fig. 37, let A represent the pitch line of a rack, and B and C the pitch circles of two wheels, then the generating circle would be rolled within B, as at 1, for the flank curves, and without it, as at 2, for the face curves of B. It would be rolled without the pitch line, as at 3, for the rack faces, and within it, as at 4, for the rack flanks, and without C, as at 5, for the faces, and within it, as at 6, for flanks of the teeth on C, and all the teeth will work correctly together however they be placed; thus C might receive motion from the rack, and B receive motion from C. Or if any number of different diameters of wheels are used they will all work correctly together and interchange perfectly, with the single condition that the same size of generating circle be used throughout. But the curves of the teeth so formed will not be alike. Thus in Fig. 38 are shown three teeth, all struck with the same size of generating circle, D being for a wheel of 12 teeth, E for a wheel of 50 teeth, and F a tooth of a rack; teeth E, F, being made wider so as to let the curves show clearly on each side, it being obvious that since the curves are due to the relative sizes of the pitch and generating circles they are equally applicable to any pitch or thickness of teeth on wheels having the same diameters of pitch circle.

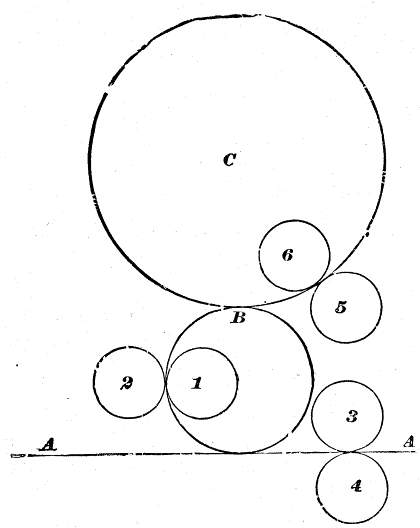

Fig. 37.

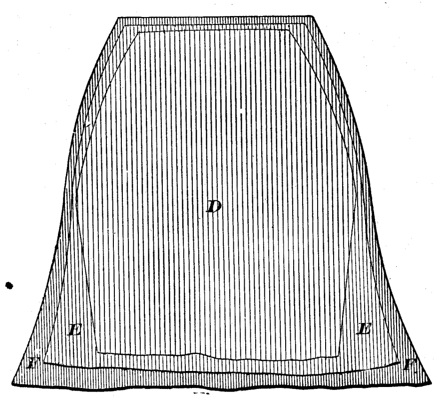

Fig. 38.

In determining the diameter of a generating circle for a set or train of wheels, we have the consideration that the smaller the diameter of the generating circle in proportion to that of the

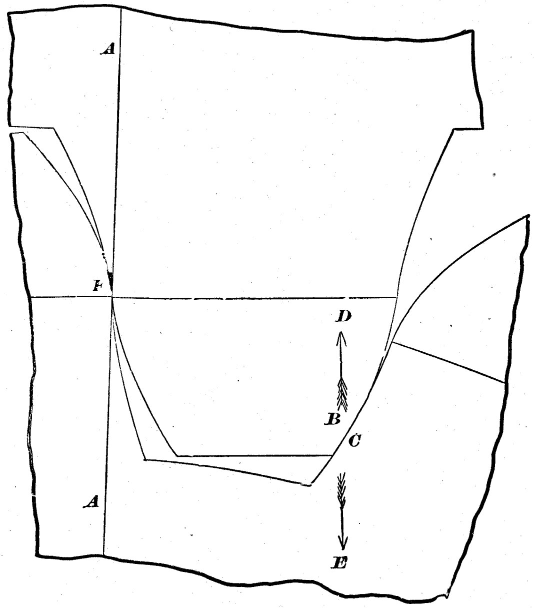

Fig. 39.

pitch circle the more the teeth are spread at the roots, and this creates a pressure tending to thrust the wheels apart, thus causing the axle journals to wear. In Fig. 39, for example, A A

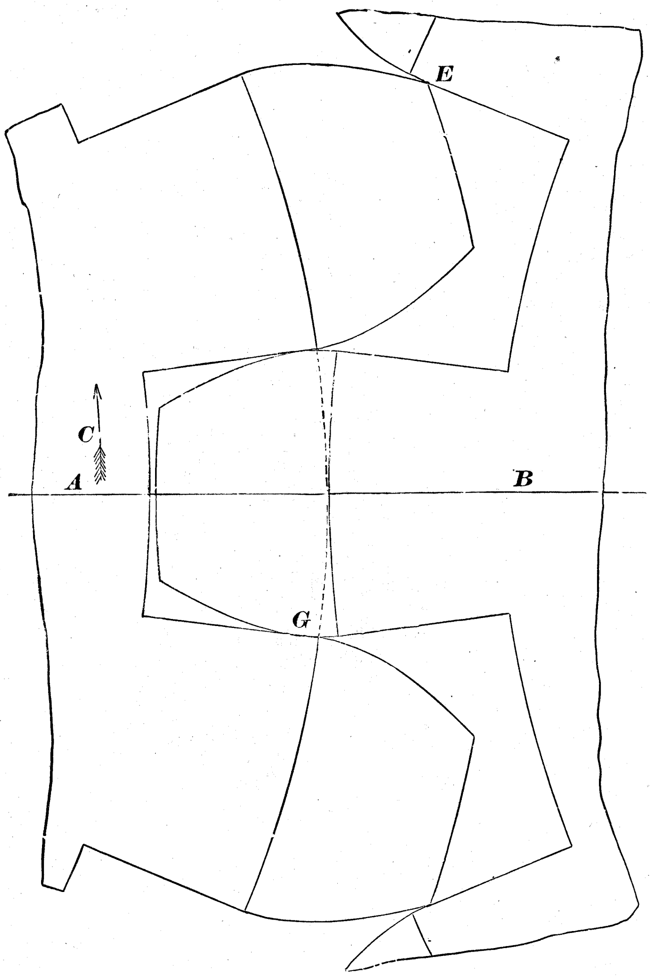

Fig. 40.

is the line of centres, and the contact of the curves at B C would cause a thrust in the direction of the arrows D, E. This thrust would exist throughout the whole path of contact save at the point F, on the line of centres. This thrust is reduced in proportion as the diameter of the generating circle is increased; thus in Fig. 40, is represented a pair of pinions of 12 teeth and 3 inch pitch, and C being the driver, there is contact at E, and at G, and E being a radial line, there is obviously a minimum of thrust.

What is known as the Willis system for interchangeable gearing, consists of using for every pitch of the teeth a generating circle whose diameter is equal to the radius of a pinion having 12 teeth, hence the pinion will in each pitch have radial flanks, and the roots of the teeth will be more spread as the number of teeth in the wheel is increased. Twelve teeth is the least number that it is considered practicable to use; hence it is obvious that under this system all wheels of the same pitch will work correctly together.

Unless the faces of the teeth and the flanks with which they work are curves produced from the same size of generating circle, the velocity of the teeth will not be uniform. Obviously the revolu-

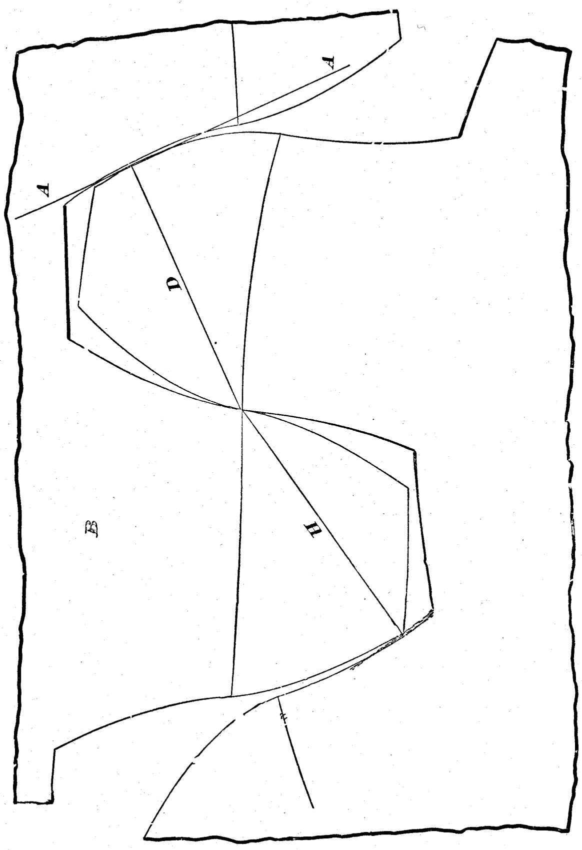

Fig. 41.

tions of the wheels will be proportionate to their numbers of teeth; hence in a pair of wheels having an equal number of teeth, the revolutions will per force be equal, but the driver will not impart uniform motion to the driven wheel, but each tooth will during the path of contact move irregularly.

The velocity of a pair of wheels will be uniform at each instant of time, if a line normal to the surfaces of the curves at their point of contact passes through the point of contact of the pitch circles on the line of centres of the wheels. Thus in Fig. 41, the line A A is tangent to the teeth curves where they touch, and D at a right angle to A A, and meets it at the point of the tooth curves, hence it is normal to the point of contact, and as it meets the pitch circles on the line of centres the velocity of the wheels will be uniform.

The amount of rolling motion of the teeth one upon the other while passing through the path of contact, will be a minimum when the tooth curves are correctly formed according to the rules given. But furthermore the sliding motion will be increased in proportion as the diameter of the generating circle is increased, and the number of teeth in contact will be increased because the arc, or path, of contact is longer as the generating circle is made larger.

Thus in Fig. 42 is a pair of wheels whose tooth curves are from a generating circle equal to the radius of the wheels, hence the of the teeth on the larger wheel B, have contact along a greater portion of their depths than do the flanks of those on the smaller, as is shown by the dotted arc I being farther from the pitch circle than the dotted arc J is, these two dotted arcs representing the

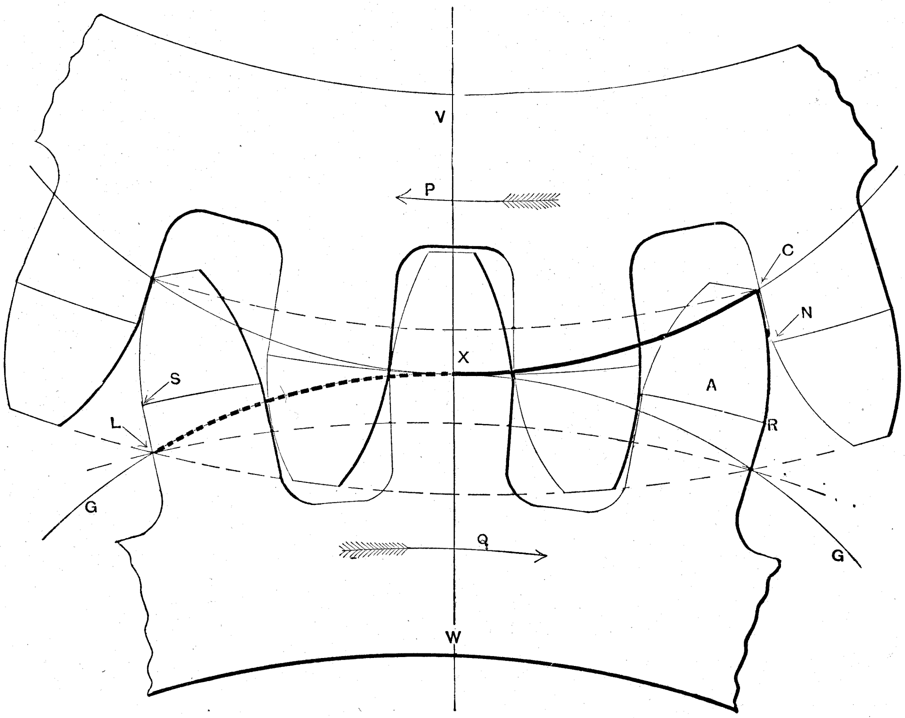

Fig. 42.

flanks are radial. The teeth are made of unusual depth to keep the lines in the engraving clear. Suppose V to be the driver, W the driven wheel or follower, and the direction of motion as at P, contact upon tooth A will begin at C, and while A is passing to the line of centres the path of contact will pass along the thickened line to X. During this time the whole length of face from C to R will have had contact with the length of flank from C to N, and it follows that the length of face on A that rolled on C N can only equal the length of C N, and that the amount of sliding motion must be represented by the length of R N on A, and the amount of rolling motion by the length N C. Again, during the arc of recess (marked by dots) the length of flank that will have had contact is the depth from S to L, and over this depth the full length of tooth face on wheel V will have swept, and as L S equals C N, the amount of rolling and of sliding motion during the arc of recess is equal to that during the arc of approach, and the action is in both cases partly a rolling and partly a sliding one. The two wheels are here shown of the same diameter, and therefore contain an equal number of teeth, hence the arcs of approach and of recess are equal in length, which will not be the case when one wheel contains more teeth than the other. Thus in Fig. 43, let A represent a segment of a pinion, and B a segment of a spur-wheel, both segments being blank with their pitch circles, the tooth height and depth being marked by arcs of circles. Let C and D represent the generating circles shown in the two respective positions on the line of centres. Let pinion A be the driver moving in the direction of P, and the arc of approach will be from E to X along the thickened arc, while the arc of recess will be as denoted by the dotted arc from X to F. The distance E X being greater than distance X F, therefore the arc of approach is longer than that of recess.

But suppose B to be the driver and the reverse will be the case, the arc of approach will begin at G and end at X, while the arc of recess will begin at X and end at H, the latter being farther from the line of centres than G is. It will be found also that, one wheel being larger than the other, the amount of sliding and rolling contact is different for the two wheels, and that the flanks

paths of the lowest points of flank contact, points F and G, marking the initial lowest contact for the two directions of revolution.

Thus it appears that there is more sliding action upon the teeth of the smaller than upon those of the larger wheel, and this is a condition that will always exist.

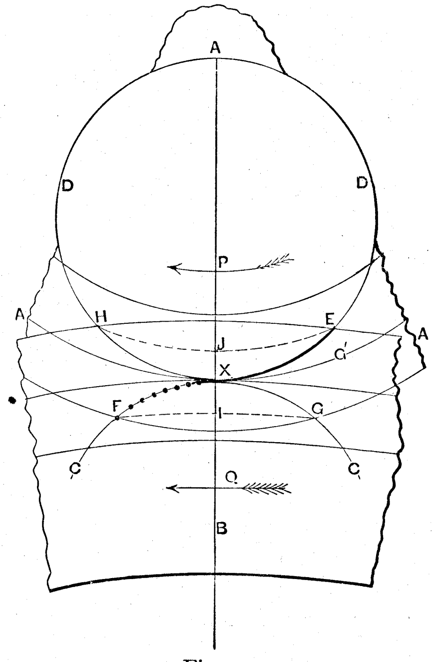

Fig. 43.

In Fig. 44 is represented portion of a pair of wheels corresponding to those shown in Fig. 42, except that in this case the diameter of the generating circle is reduced to one quarter that of the pitch diameter of the wheels. V is the driver in the direction of P, and contact will begin at C; hence the depth of flank on the teeth of V that will have contact is C N, which, the wheels, being of equal diameter, will remain the same whichever wheel be the driver, and in whatever direction motion occurs. The amount of rolling motion is, therefore, C N, and that of sliding is the difference between the distance C N and the length of the tooth face.

If now we examine the distance C N in Fig. 42, we find that train of gearing in which the generating circle equals the radius of the pinion, the pinion will wear out of shape the quickest, and the largest wheel the least; because not only does each tooth on the pinion more frequently come into action on account of its increased revolutions, but furthermore the length of flank that has contact is less, while the amount of sliding action is greater. In Fig. 45, for example, are a wheel and pinion, the latter having radial flanks and the pinion being the driver, the arc of approach

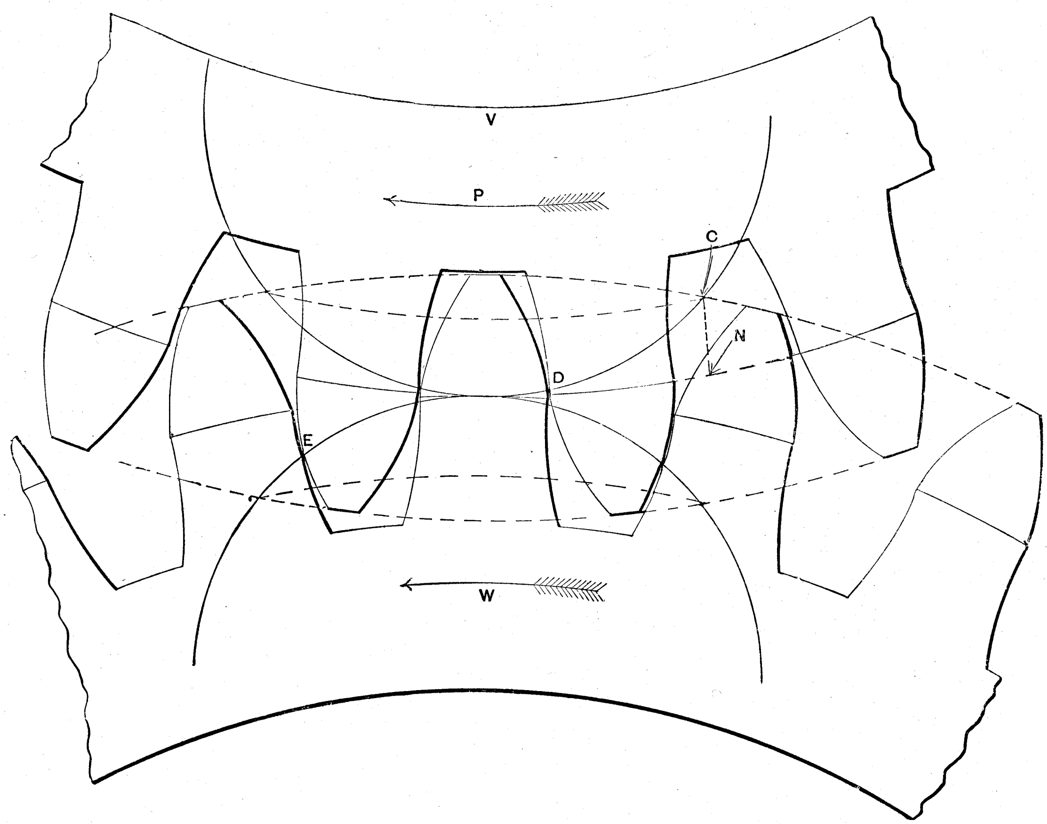

Fig. 44.

reducing the diameter of generating circle in Fig. 44 has increased the depth of flank that has contact, and therefore increased the rolling motion of the tooth face along the flank, and correspondingly diminished the sliding action of the tooth contact. But at the same time we have diminished the number of teeth in contact. Thus in Fig. 42 there are three teeth in driving contact, while in Fig. 44 there are but two, viz., D and E.

In an article by Professor Robinson, attention is called to the

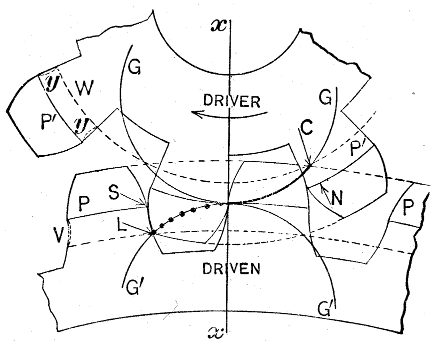

Fig. 45.

fact that if the teeth of wheels are not formed to have correct curves when new, they cannot be improved by wear; and this will be clearly perceived from the preceding remarks upon the amount of rolling and sliding contact. It will also readily appear that the nearer the diameter of the generating to that of the base circle the more the teeth wear out of correct shape; hence, in a

is the thickened arc from C to the line of centres, while the arc of recess is denoted by the dotted arc. As contact on the pinion flank begins at point C and ends at the line of centres, the total depth of flank that suffers wear from the contact is that from C to N; and as the whole length of the wheel tooth face sweeps over this depth C N, the pinion flanks must wear faster than the wheel faces, and the pinion flanks will wear underneath, as denoted by the dotted curve on the flanks of tooth W. In the case of the wheel, contact on its tooth flanks begins at the line of centres and ends at L, hence that flank can only wear between point L and the pitch line S; and as the whole length of pinion face sweeps on this short length L S, the pinion flank will wear most, the wear being in the direction of the dotted arc on the left-hand side V of the tooth. Now the pinion flank depth C N, being less than the wheel flank depth S L, and the same length of tooth face sweeping (during the path of contact) over both, obviously the pinion tooth will wear the most, while both will, as the wear proceeds, lose their proper flank curve. In Fig. 46 the generating arcs, G and G′, and the wheel are the same, but the pinion is larger. As a result the acting length C N, of pinion flank is increased, as is also the acting length S L, of wheel flank; hence, the flanks of both wheels would wear better, and also better preserve their correct and original shapes.

It has been shown, when referring to Figs. 42 and 44, when treating of the amount of sliding and of rolling motion, that the smaller the diameter of rolling circle in proportion to that of pitch circle, the longer the acting length of flank and the more the amount of rolling motion; and it follows that the teeth would also preserve their original and true shape better. But the wear of the teeth, and the alteration of tooth form by reason of that wear, will, in any event, be greater upon the pinion than upon the wheel, and can only be equal when the two wheels are of equal diameter, in which case the tooth curves will be alike on both wheels, and the acting depths of flank will be equal, as shown in Fig. 47, the flanks being radial, and the acting depths of flank being shown at J K. In Fig. 48 is shown a pair of wheels with a generating circle, G and G′, of one quarter the diameter of the base circle or pitch diameter, and the acting length of flank is

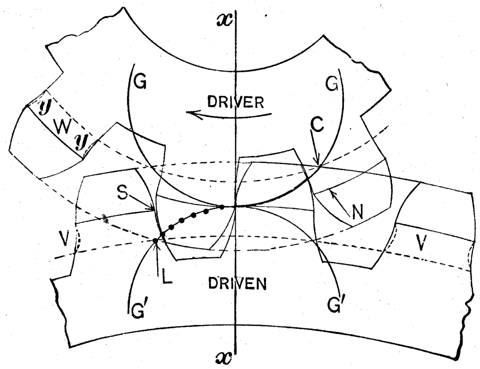

Fig. 46.

shown at L M. The wear of the teeth would, therefore, in this latter case, cause it in time to assume the form shown in Fig. 49. But it is to be noted that while the acting depth of flank has been increased the arcs of contact have been diminished, and that in Fig. 47 there are two teeth in contact, while in Fig. 48 there is but one, hence the pressure upon each tooth is less in proportion as the diameter of the generating circle is increased. If a train of wheels are to be constructed, or if the wheels are to be capable

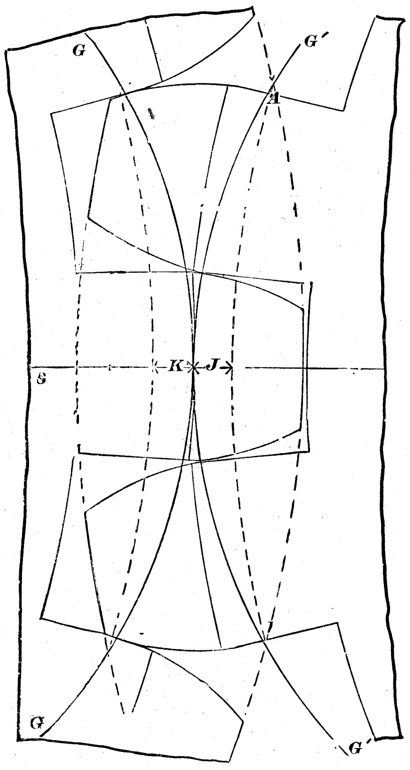

Fig. 47.

of interchanging with other combinations of wheels of the same pitch, the diameter of the generating circle must be equal to the smallest wheel or pinion, which is, under the Willis system, a pinion of 12 teeth; under the Pratt and Whitney, and Brown and Sharpe systems, a pinion of 15 teeth.

But if a pair or a particular train of gears are to be constructed, then a diameter of generating circle may be selected that is considered most suitable to the particular conditions; as, for example, it may be equal to the radius of the smallest wheel giving it radial flanks, or less than that radius giving parallel or spread flanks. But in any event, in order to transmit continuous motion, the diameter of generating circle must be such as to give arcs of action that are equal to the pitch, so that each pair of teeth will come into action before the preceding pair have gone out of action.

It may now be pointed out that the degrees of angle that the teeth move through always exceeds the number of degrees of

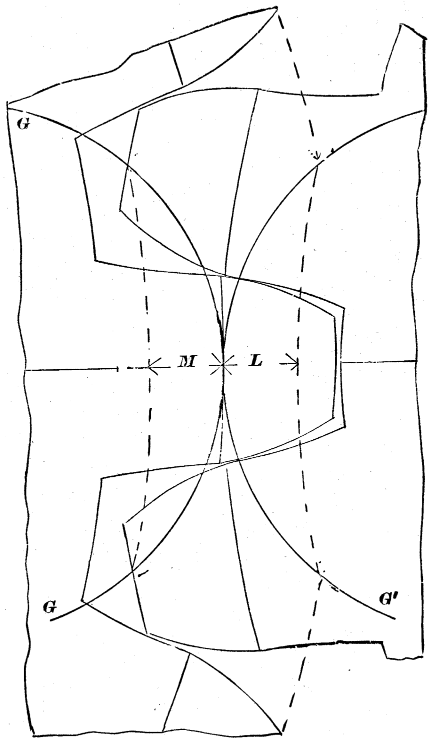

Fig. 48.

angle contained in the paths of contact, or, in other words, exceeds the degrees contained in the arcs of approach and recess combined.

In Fig. 50, for example, are a wheel A and pinion B, the teeth on the wheel being extended to a point. Suppose that the wheel A is the driver, and contact will begin between the two teeth D and F on the dotted arc. Now suppose tooth D to have moved to position C, and F will have been moved to position H. The

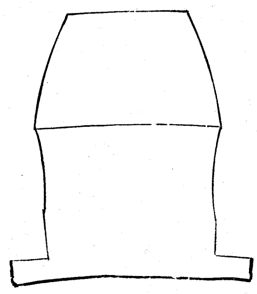

Fig. 49.

degrees of angle the pinion has been moved through are therefore denoted by I, whereas the degrees of angle the arcs of contact contain are therefore denoted by J.

The degrees of angle that the wheel A has moved through are obviously denoted by E, because the point of tooth D has during the arcs of contact moved from position D to position C. The degrees of angle contained in its path of contact are denoted by K, and are less than E, hence, in the case of teeth terminating in a point as tooth D, the excess of angle of action over path of contact is as many degrees as are contained in one-half the thickness of the tooth, while when the points of the teeth are cut off, the excess is the number of degrees contained in the distance between the corner and the side of the tooth as marked on a tooth at P.

With a given diameter of pitch circle and pitch diameter of wheel, the length of the arc of contact will be influenced by the height of the addendum from the pitch circle, because, as has been shown, the arcs of approach and of recess, respectively, begin and end on the addendum circle.

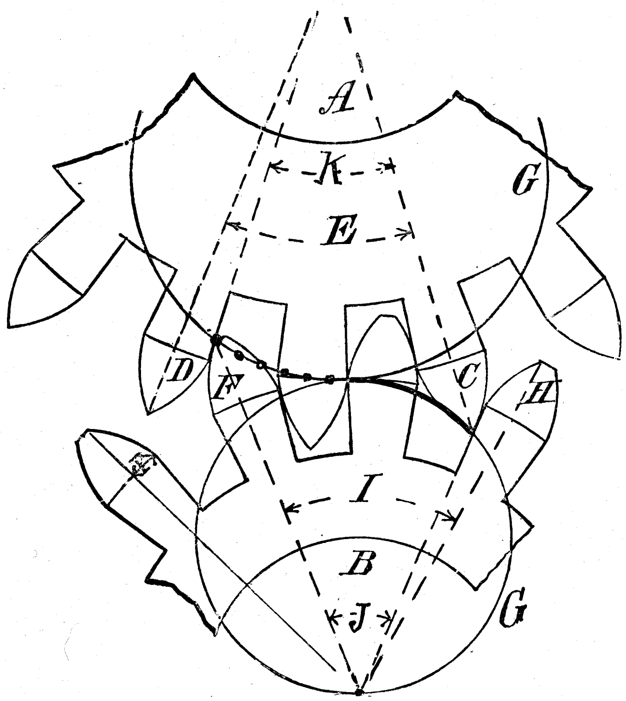

Fig. 50.

If the height of the addendum on the follower be reduced, the arc of approach will be reduced, while the arc of recess will not be altered; and if the follower have no addendum, contact between the teeth will occur on the arc of recess only, which gives a smoother motion, because the action of the driver is that of dragging rather than that of pushing the follower. In this case, however, the arc of recess must, to produce continuous motion, be at least equal to the pitch.

It is obvious, however, that the follower having no addendum would, if acting as a driver to a third wheel, as in a train of wheels, act on its follower, or the fourth wheel of the train, on the arc of approach only; hence it follows that the addendum might be reduced to diminish, or dispensed with to eliminate action, on the arc of approach in the follower of a pair of wheels only, and not in the case of a train of wheels.

To make this clear to the reader it may be necessary to refer again to Fig. 33 or 34, from which it will be seen that the action of the teeth of the driver on the follower during the arc of approach is produced by the flanks of the driver on the faces of the follower. But if there are no such faces there can be no such contact.

On the arc of recess, however, the faces of the driver act on the flanks of the follower, hence the absence of faces on the follower is of no import.

From these considerations it also appears that by giving to the driver an increase of addendum the arc of recess may be increased without affecting the arc of approach. But the height of addendum in machinists' practice is made a constant proportion of the pitch, so that the wheel may be used indiscriminately, as circumstances may require, as either a driver or a follower, the arcs of approach and of recess being equal. The height of addendum, however, is an element in determining the number of teeth in contact, and upon small pinions this is of importance.

In Fig. 51, for example, is shown a section of two pinions of equal diameters, and it will be observed that if the full line A determined the height of the addendum there would be contact either at C or B only (according to the direction in which the motion took place).

With the addendum extended to the dotted circle, contact would be just avoided, while with the addendum extended to D there would be contact either at E or at F, according to which direction the wheel had motion.

This, by dividing the strain over two teeth instead of placing it all upon one tooth, not only doubles the strength for driving capacity, but decreases the wear by giving more area of bearing surface at each instant of time, although not increasing that area in proportion to the number of teeth contained in the wheel.

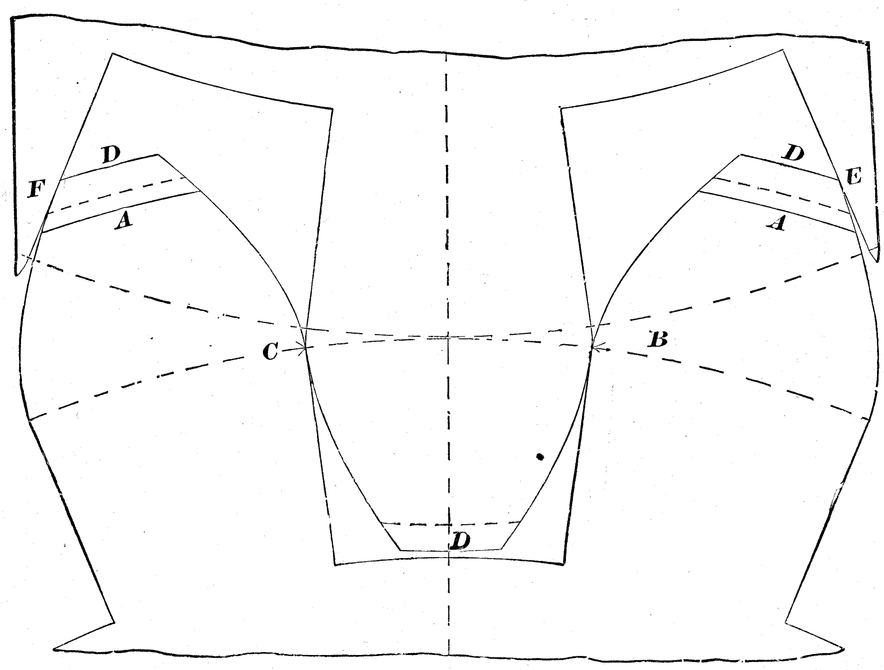

Fig. 51.

In wheels of larger diameter, short teeth are more permissible, because there are more teeth in contact, the number increasing with the diameters of the wheels. It is to be observed, however, that from having radial flanks, the smallest wheel is always the weakest, and that from making the most revolutions in a given time, it suffers the most from wear, and hence requires the greatest attainable number of teeth in constant contact at each period of time, as well as the largest possible area of bearing or wearing surface on the teeth.

It is true that increasing the "depth of tooth to pitch line" increases the whole length of tooth, and, therefore, weakens it; but this is far more than compensated for by distributing the strain over a greater number of teeth. This is in practice accomplished, *when circumstances will permit*, by making the

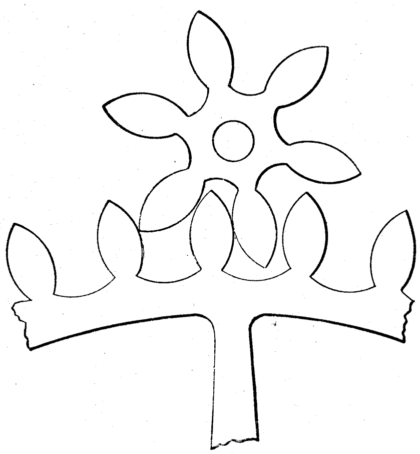

Fig. 52.

pitch finer, giving to a wheel, of a given diameter, a greater number of teeth.

When the wheels are required to transmit motion rather than power (as in the case of clock wheels), to move as frictionless as possible, and to place a minimum of thrust on the journals of the shafts of the wheels, the generating circle may be made nearly as large as the diameter of the pitch circle, producing teeth of the form shown in Fig. 52. But the minimum of friction is attained when the two flanks for the tooth are drawn into one common hypocycloid, as in Fig. 53. The difference between the form of tooth shown in Fig. 52 and that shown in Fig. 53, is merely due to an increase in the diameter of the generating circle for the latter. It will be observed that in these forms the

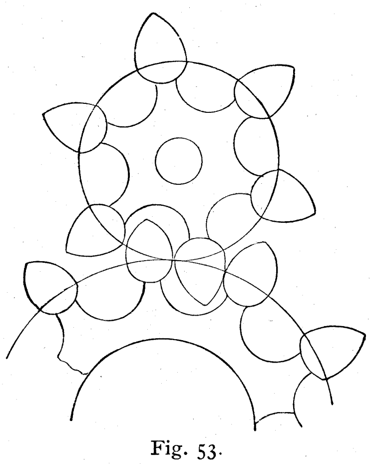

Fig. 53.

acting length of flank diminishes in proportion as the diameter of the generating circle is increased, the ultimate diameter of generating circle being as large as the pitch circles.

* This form is undesirable in that there is contact on one side only (on the arc of approach) of the line of centres, but the flanks of the teeth may be so modified as to give contact on the arc of recess also, by forming the flanks as shown in Fig. 54, the flanks, or rather the parts within the pitch circles, being nearly half circles, and the parts without with peculiarly formed faces, as shown in the figure. The pitch circles must still be regarded as the rolling circles rolling upon each other. Suppose *b* a tracing point on B, then as B rolls on A it will describe the epicycloid *a b*.

* From an article by Professor Robinson.

A parallel line *c d* will work at a constant distance as at *c d* from *a b*, and this distance may be the radius of that part of D that is within the pitch line, the same process being applied to the teeth on both wheels. Each tooth is thus composed of a spur based upon a half cylinder.

Comparing Figs. 53 and 54, we see that the bases in 53 are flattest, and that the contact of faces upon them must range

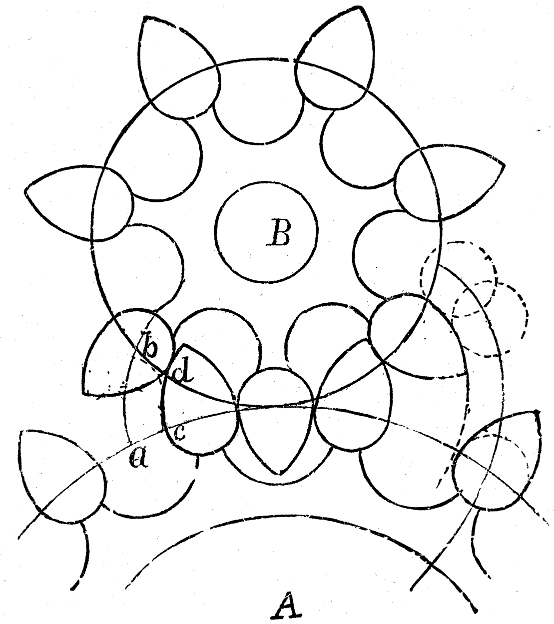

Fig. 54.

nearer the pitch line than in 54. Hence, 53 presents a more favorable obliquity of the line of direction of the pressures of tooth upon tooth. In seeking a still more favorable direction by going outside for the point of contact, we see by simply recalling the method of generating the tooth curves, that tooth contacts outside the pitch lines have no possible existence; and hence, Fig. 53 may be regarded as representing that form of toothed gear which will operate with less friction than any other known form.

This statement is intended to cover fixed teeth only, and not that complicated form of the trundle wheel in which the cylinder

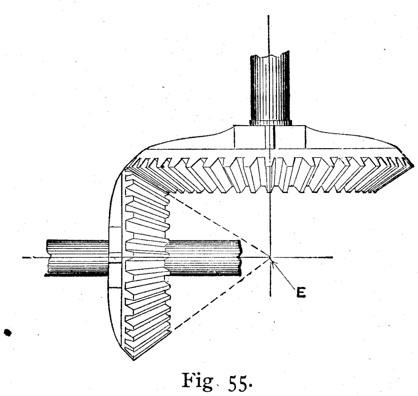

Fig. 55.

teeth are friction rollers. No doubt such would run still easier, even with their necessary one-sided contacts. Also, the statement is supposed to be confined to such forms of teeth as have good practical contacts at and near the line of centres.

Bevel-gear wheels are employed to transmit motion from one shaft to another when the axis of one is at an angle to that of the other. Thus in Fig. 55 is shown a pair of bevel-wheels to transmit motion from shafts at a right angle. In bevel-wheels all the lines of the teeth, both at the tops or points of the teeth, at the bottoms of the spaces, and on the sides of the teeth, radiate from the centre E, where the axes of the two shafts would meet if produced. Hence the depth, thickness, and height of the tooth decreases as the point E is approached from the diameter of the wheel, which is always measured on the pitch circle at the largest end of the cone, or in other words, at the largest pitch diameter.

The principles governing the practical construction of the curves for the teeth of the bevel-wheels may be explained as follows:—

In Fig. 56 let F and G represent two shafts, rotating about their respective axes; and having cones whose greatest diameters are at A and B, and whose points are at E. The diameter A being

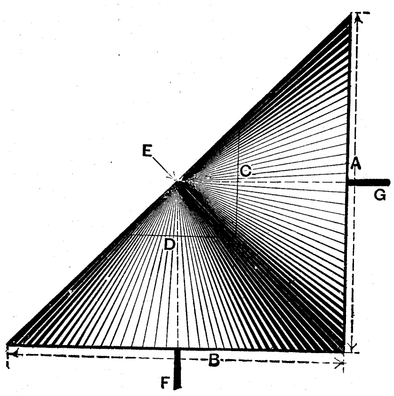

Fig. 56.

equal to that of B their circumferences will be equal, and the angular and velocity ratios will therefore be equal.

Let C and D represent two circles about the respective cones, being equidistant from E, and therefore of equal diameters and circumferences, and it is obvious that at every point in the length of each cone the velocity will be equal to a point upon the other so long as both points are equidistant from the points of intersection of the axes of the two shafts; hence if one cone drive the other by frictional contact of surfaces, both shafts will be rotated at an equal speed of rotation, or if one cone be fixed and the other moved around it, the contact of the surfaces will be a rolling contact throughout. The line of contact between the two cones will be a straight line, radiating at all times from the point E. If such, however, is not the case, then the contact will no longer be a rolling one. Thus, in Fig. 57 the diameters or circumferences at A and B being equal, the surfaces would roll upon each other, but on account of the line of contact not radiating from E (which is the common centre of motion for the two

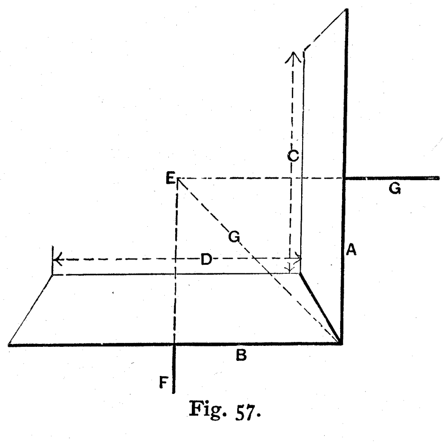

Fig. 57.

shafts) the circumference C is less than that of D, rendering a rolling contact impossible.

We have supposed that the diameters of the cones be equal, but the conditions will remain the same when their diameters are unequal; thus, in Fig. 58 the circumference of A is twice that of B, hence the latter will make two rotations to one of the former, and the contact will still be a rolling one. Similarly the circumference of D is one half that of C, hence D will also make two rotations to one of C, and the contact will also be a rolling one; a condition which will always exist independent of the diameters of the wheels so long as the angles of the faces, or wheels, or (what is the same thing, the line of contact between the two,) radiates from the point E, which is located where the axes of the shafts would meet.

The principles governing the forms of the cones on which the teeth are to be located thus being explained, we may now consider the curves of the teeth. Suppose that in Fig. 59 the cone A is fixed, and that the cone whose axis is F be rotated upon it in the direction of the arrow. Then let a point be fixed in any part

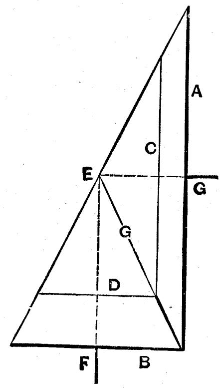

Fig. 58.

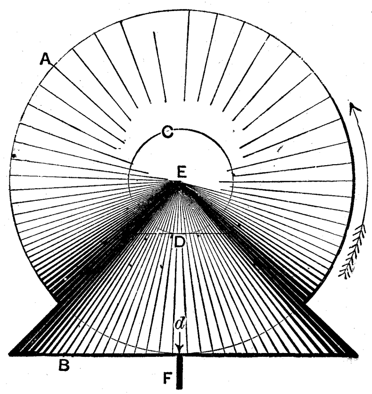

Fig. 59.

of the circumference of B (say at *d*), and it is evident that the path of this point will be as B rolls around the axis F, and at the same time around A from the centre of motion, E. The curve so generated or described by the point *d* will be a spherical epicycloid. In this case the exterior of one cone has rolled upon the coned surface of the other; but suppose it rolls upon the interior, as around the walls of a conical recess in a solid body; then a point in its circumference would describe a curve known as the

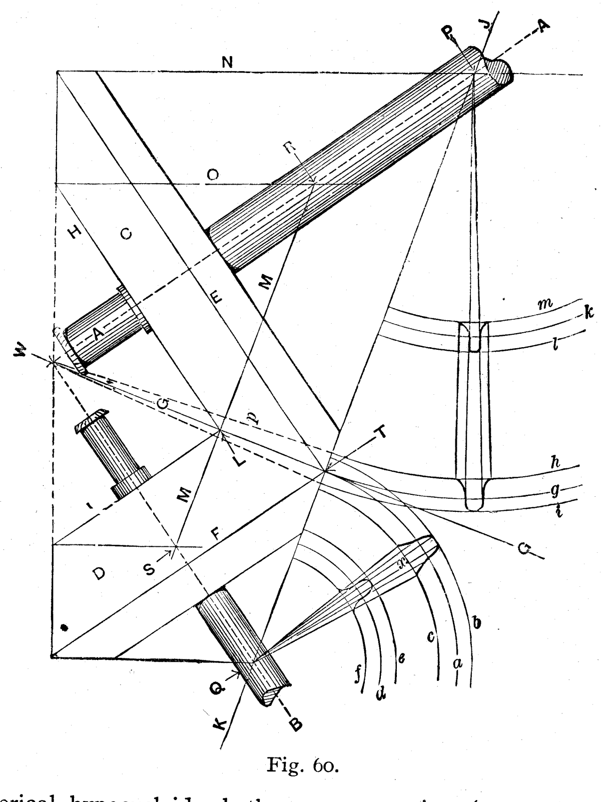

Fig. 60.

spherical hypocycloid; both curves agreeing (except in their spherical property) to the epicycloid and hypocycloid of the spur-wheel. But this spherical property renders it very difficult indeed to practically delineate or mark the curves by rolling contact, and on account of this difficulty Tredgold devised a method of construction whereby the curves may be produced sufficiently accurate for all practical purposes, as follows:—

In Fig. 60 let A A represent the axis of one shaft, and B the axis of the other, the axes of the two meeting at W. Mark E, representing the diameter of one wheel, and F that of the other (both lines representing the pitch circles of the respective wheels). Draw the line G G passing through the point W, and the point T, where the pitch circles E, F meet, and G G will be the line of contact between the cones. From W as a centre, draw on each side of G G dotted lines as *p*, representing the height of the teeth above and below the pitch line G G. At a right angle to G G mark the line J K, and from the junction of this line with axis B (as at Q) as a centre, mark the arc *a*, which will represent the pitch circle for the large diameter of pinion D; mark also the arc *b* for the addendum and *c* for the roots of the teeth, so that from *b* to *c* will represent the height of the tooth at that end.

Similarly from P, as a centre, mark (for the large diameter of wheel C,) the pitch circle *g*, root circle *h*, and addendum *i*. On these arcs mark the curves in the same manner as for spur-wheels. To obtain these arcs for the small diameters of the wheels, draw M M parallel to J K. Set the compasses to the radius R L, and from P, as a centre, draw the pitch circle *k*. To obtain the depth for the tooth, draw the dotted line *p*, meeting the circle *h*, and the point W. A similar line from circle *i* to W will show the height of the addendum, or extreme diameter; and mark the tooth curves on *k*, *l*, *m*, in the same manner as for a spur-wheel.

Similarly for the pitch circle of the small end of the pinion teeth, set the compasses to the radius S L, and from Q as a centre, mark the pitch circle *d*, outside of *d* mark *e* for the height of the addendum and inside of *d* mark *f* for the roots of the teeth at that end. The distance between the dotted lines (as *p*) represents the full height of the teeth, hence *h* meets line *p*, being the root of tooth for large wheel, and to give clearance, the point of the pinion teeth is marked below, thus arc *b* does not meet *h* or *p*. Having obtained these arcs the curves are rolled as for a spur-wheel.

A tooth thus marked out is shown at *x*, and from its curves between *b c*, a template for the large diameter of the pinion tooth may be made, while from the tooth curves between the arcs *e f*, a template for the smallest tooth diameter of the pinion can be made.

Similarly for the wheel C the outer end curves are marked on the lines *g*, *h*, *i*, and those for the inner end on the lines *k*, *l*, *m*.

Internal or annular gear-wheels have their tooth curves formed by rolling the generating circle upon the pitch circle or base circle, upon the same general principle as external or spur-wheels. But the tooth of the annular wheel corresponds with the space in the spur-wheel, as is shown in Fig. 61, in which curve A forms the flank of a tooth on a spur-wheel P, and the face of a tooth on the annular wheel W. It is obvious then that the generating circle is rolled within the pitch circle for the face of the wheel and without for its flank, or the reverse of the process for spur-wheels. But in the case of internal or annular wheels the path of contact of tooth upon tooth with a pinion having a given number of teeth increases in proportion as the number of teeth in the wheel is diminished, which is also the reverse of what occurs in spur-wheels; as will readily be perceived when it is considered that if in an internal wheel the pinion have as many teeth as the wheel the contact would exist around the whole pitch circles of the wheel and pinion and the two would rotate together without any motion of tooth upon tooth. Obviously then we have, in the case of internal wheels, a consideration as to what is the greatest number (as well as what is the least number) of teeth a pinion may contain to work with a given wheel, whereas in spur-wheels the reverse is again the case, the consideration being how few teeth the wheel may contain to work with a given pinion. Now it is found that although the curves of the teeth in internal wheels and pinions may be rolled according to the principles already laid down for spur-wheels, yet cases may arise in which internal gears will not work under conditions in which spur-wheels would work, because the internal wheels will not engage together. Thus, in Fig. 62, is a pinion of 12 teeth and a wheel of 22 teeth, a generating circle having a diameter equal to the radius of the pinion having been used for all the tooth curves of both wheel and pinion.

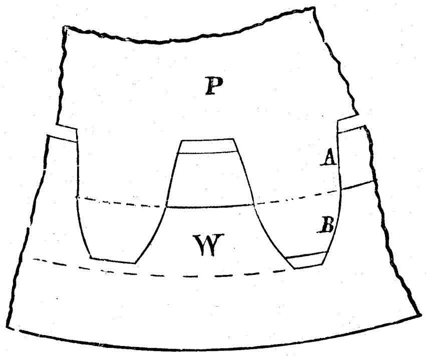

Fig. 61.

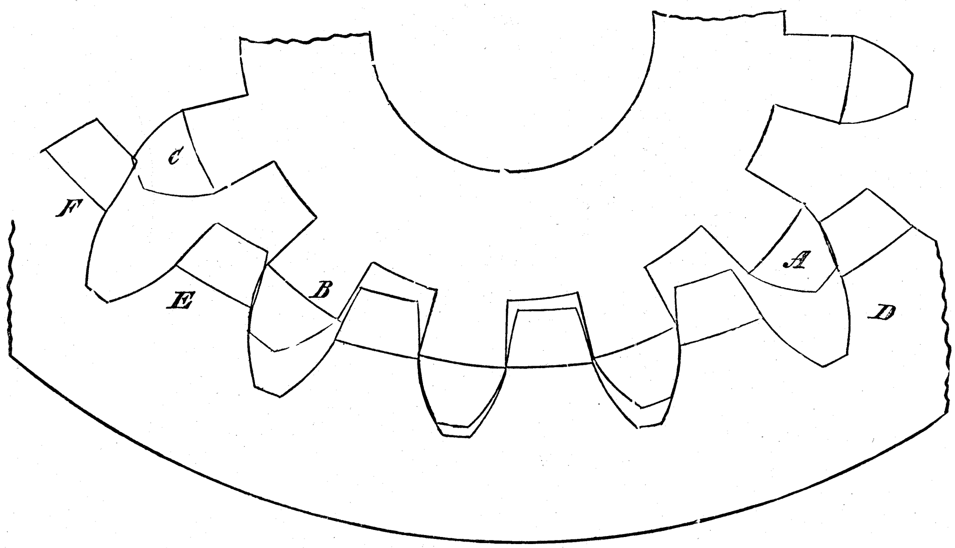

Fig. 62.

It will be observed that teeth A, B, and C clearly overlap teeth D, E, and F, and would therefore prevent the wheels from engaging to the requisite depth.

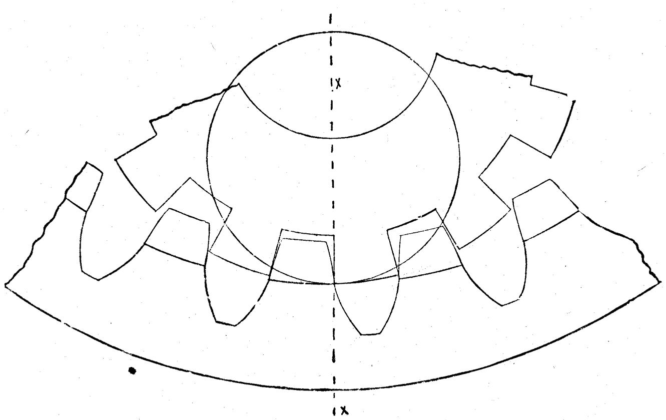
Fig. 63.

This may of course be remedied by taking the faces off the pinion, as in Fig. 63, and thus confining the arc of contact to an arc of recess if the pinion drives, or an arc of approach if the wheel drives; or the number of teeth in the pinion may be reduced, or that in the wheel increased; either of which may be carried out to a degree sufficient to enable the teeth to engage and not interfere one with the other. In Fig. 64 the number of teeth in the pinion P is reduced from 12 to 6, the wheel W having 22 as before, and it will be observed that the teeth engage and properly clear each other.

By the introduction into the figure of a segment of a spur-wheel also having 22 teeth and placed on the other side of the pinion, it is shown that the path of contact is greater, and therefore the angle of action is greater, in internal than in spur gearing. Thus suppose the pinion to drive in the direction of the arrows and the thickened arcs A B will be the arcs of approach, A measuring longer than B. The dotted arcs C D represent the arcs of receding contact and C is found longer than D, the angles of action being 66° for the spur-wheels and 72° for the annular wheel.

On referring again to Fig. 62 it will be observed that it is the faces of the teeth on the two wheels that interfere and will prevent them from engaging, hence it will readily occur to the mind that it is possible to form the curves of the pinion faces correct to work with the faces of the wheel teeth as well as with the flanks; or it is possible to form the wheel faces with curves that will work correctly with the faces, as well as with the flanks of the pinion teeth, which will therefore increase the angle of action, and professor McCord has shown in an article in the London *Engineering* how to accomplish this in a simple and yet exceedingly ingenious manner which may be described as follows :—

It is required to find a describing circle that will roll the

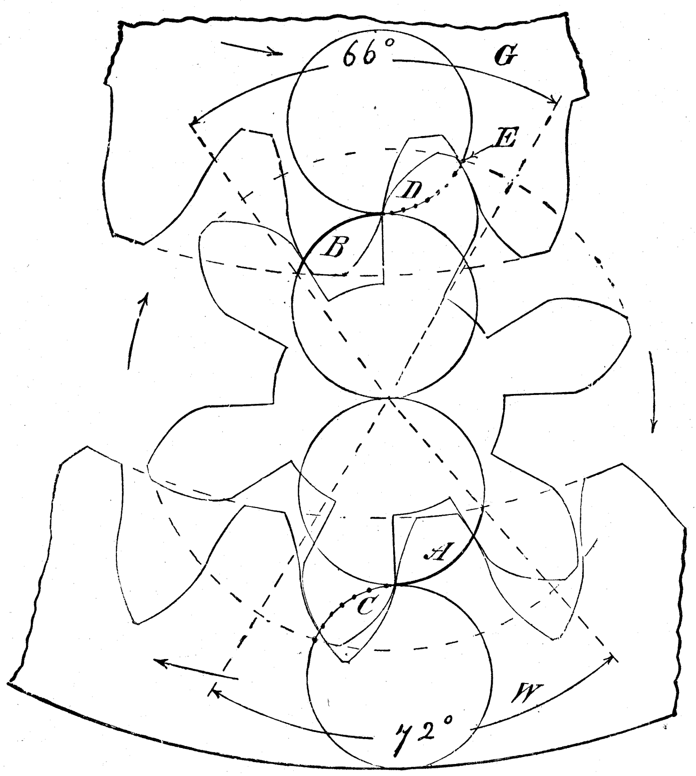

Fig. 64.

curves for the flanks of the pinion and the faces of the wheels, and also a describing circle for the flanks of the wheel and the faces of the pinion; the curve for the wheel faces to work correctly with the faces as well as with the flanks of the pinion, and the curve for the pinion faces to work correctly with both the flanks and faces of the internal wheel.

In Fig. 65 let P represent the pitch circle of an annular or internal wheel whose centre is at A, and Q the pitch circle of a pinion whose centre is at B, and let R be a describing circle whose centre is at C, and which is to be used to roll all the curves for the teeth. For the flanks of the annular wheel we may roll R within P, while for the faces of the wheel we may roll R outside of P, but in the case of the pinion we cannot roll R within Q, because R is larger than Q, hence we must find some other rolling circle of less diameter than R, and that can be used in its stead (the radius of R always being greater than the radius of the axis of the wheel and pinion for reasons that will appear presently). Suppose then that in Fig. 66 we have a ring whose bore R corresponds in diameter to the intermediate describing circle R, Fig. 65 and that Q represents the pinion. Then we may roll R around and in contact with the pinion Q, and a tracing point in R will trace the curve M N O, giving a curve a portion of which may be used for the faces of the pinion. But suppose that instead of rolling the intermediate describing circle R around P,

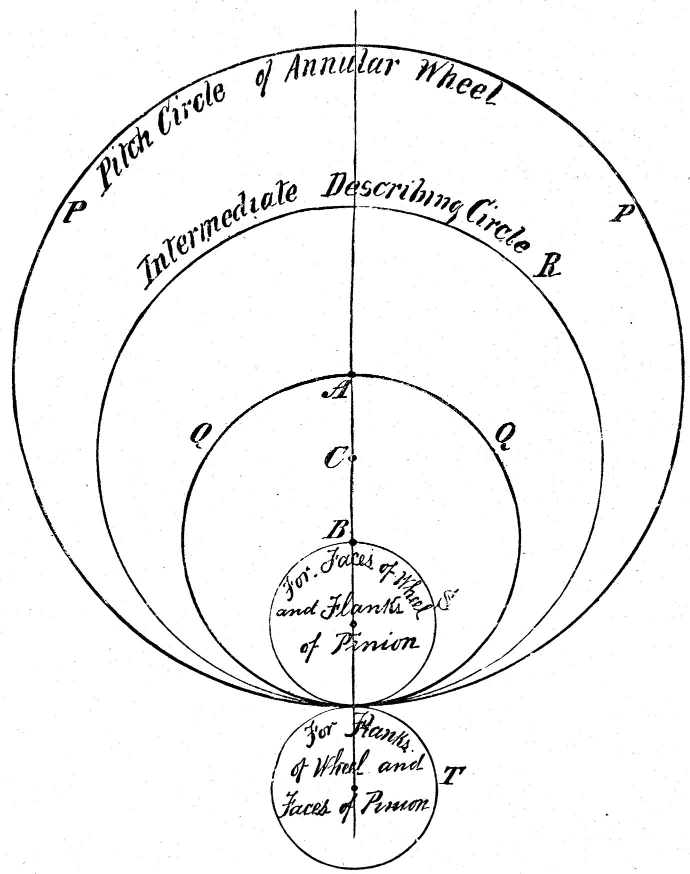

Fig. 65.

we roll the circle T around P, and it will trace precisely the same curve M N O; hence for the faces of the pinion we have found a rolling circle T which is a perfect substitute for the intermediate

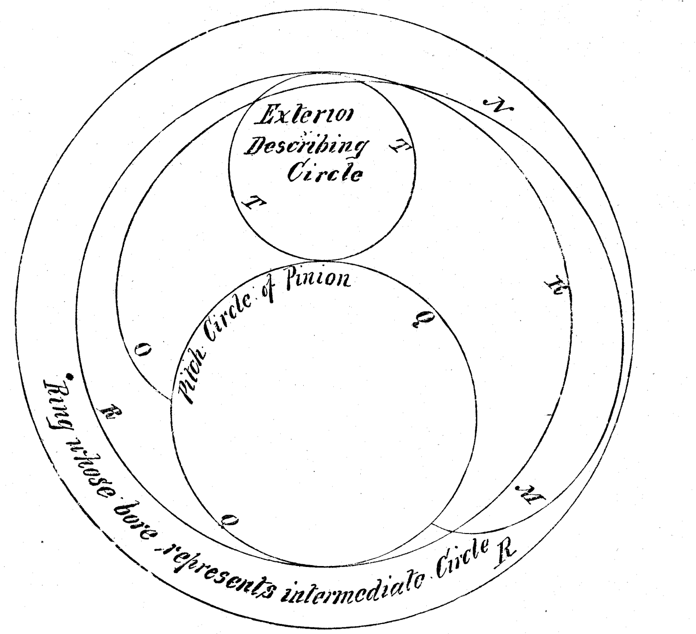

Fig. 66.

circle Q, and which it will always be, no matter what the diameters of the pinion and of the intermediate describing circle may be, providing that the diameter of T is equal to the difference between the diameters of the pinion and that of the intermediate describing circle as in the figure. If now we use this describing circle to roll the flanks of the annular wheel as well as the faces of the pinion, these faces and flanks will obviously work correctly together. Since this describing circle is rolled on the outside of dotted portion of the exterior describing circle as in ordinary gearing. But in addition there will be an arc of recess along the dotted portion of the intermediate circle R, which arc is due to the faces of the pinion acting upon the faces as well as upon the flanks of

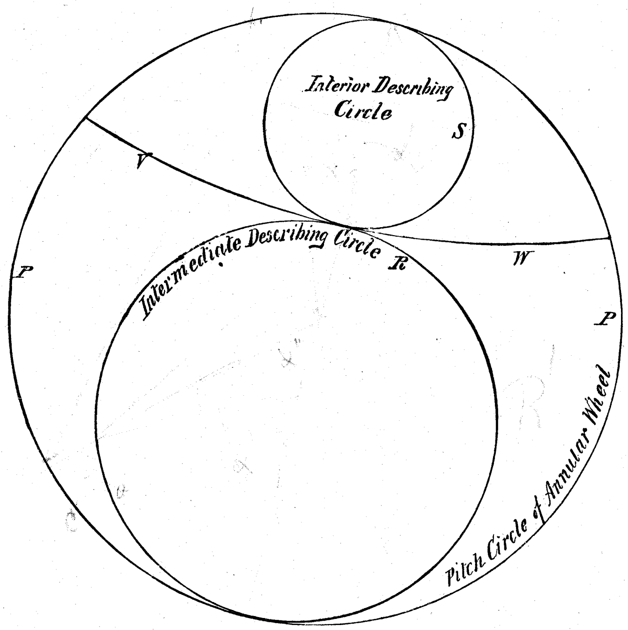

Fig. 67.

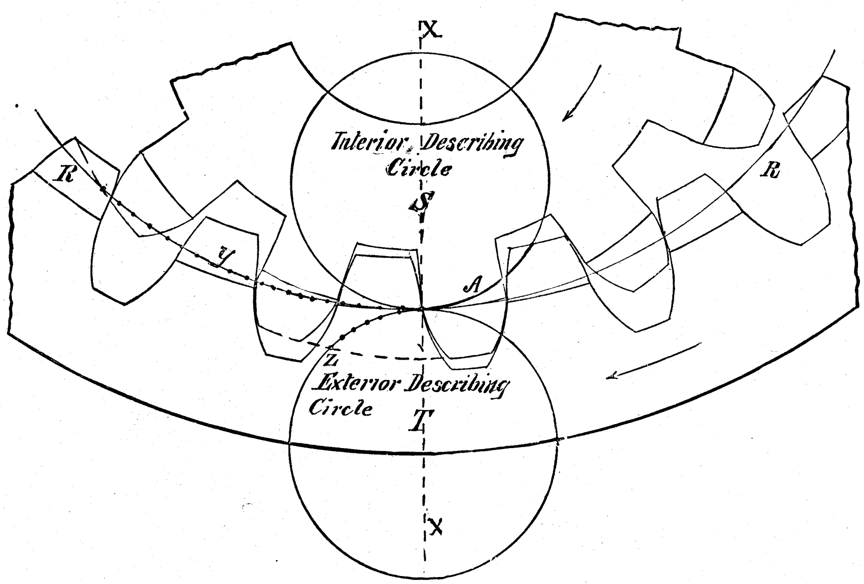

Fig. 68.

the pinion and on the outside of the annular wheel we may distinguish it as the exterior describing circle.

Now instead of rolling the intermediate describing circle R within the annular wheel P for the face curves of the teeth upon P, we may find some other circle that will give the same curve and be small enough to be rolled within the pinion Q for its teeth flanks. Thus in Fig. 67 P represents the pitch circle of the annular wheel and R the intermediate circle, and if R be rolled within P, a point on the circumference of R will trace the curve V W. But if we take the circle S, having a diameter equal to the difference between the diameter of R and that of P, and roll it within P, a point in its circumference will trace the same curve V W; hence S is a perfect substitute for R, and a portion of the curve V W may be used for the faces of the teeth on the annular wheel. The circle S being used for the pinion flanks, the wheel faces and pinion flanks will work correctly together, and as the circle S is rolled within the pinion for its flanks and within the wheel for its faces, it may be distinguished as the interior describing circle.

To prove the correctness of the construction it may be noted that with the particular diameter of intermediate describing circle used in Fig. 65, the interior and exterior describing circles are of equal diameters; hence, as the same diameter of describing circle is used for all the faces and flanks of the pair of wheels they will obviously work correctly together, in accordance with the rules laid down for spur gearing. The radius of S in Fig. 69 is equal to the radius of the annular wheel, less the radius of the intermediate circle, or the radius from A to C. The radius of the exterior describing circle T is the radius of the intermediate circle less the radius of the pinion, or radius C B in the figure.

Now the diameter of the intermediate circle may be determined at will, but cannot exceed that of the annular wheel or be less than the pinion. But having been selected between these two limits the interior and exterior describing circles derived from it give teeth that not only engage properly and avoid the interference shown in Fig. 62, but that will also have an additional arc of action during the recess, as is shown in Fig. 68, which represents the wheel and pinion shown in Fig. 62, but produced by means of the interior and exterior describing circles. Supposing the pinion to be the driver the arc of approach will be along the thickened arc of the interior describing circle, while during the arc of recess there will be an arc of contact along the the wheel teeth. It is obvious from this that as soon as a tooth passes the line of centres it will, during a certain period, have two points of contact, one on the arc of the exterior describing circle, and another along the arc of R, this period continuing

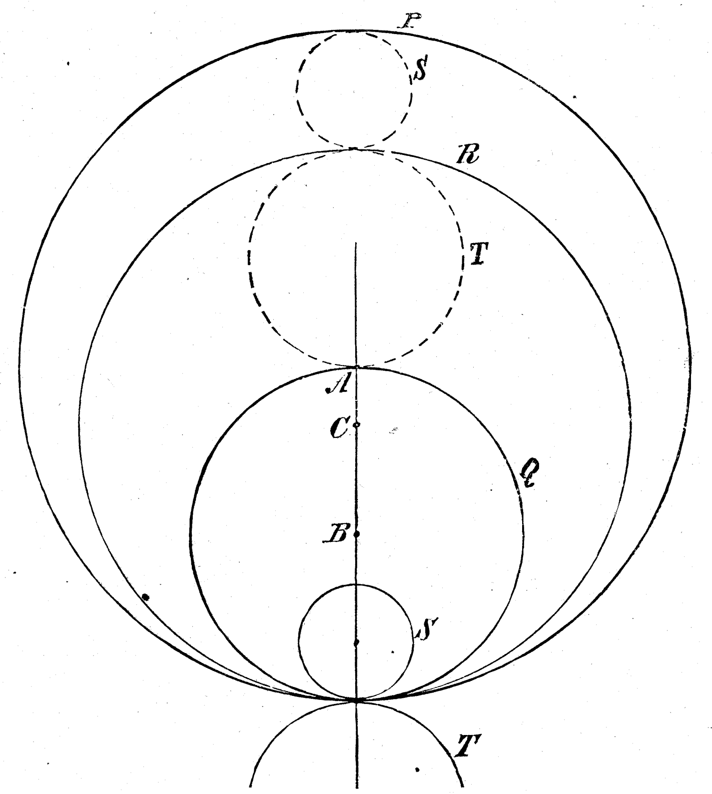

Fig. 69.

until the addendum circle of the pinion crosses the dotted arc of the exterior describing circle at Z.

The diameters of the interior and exterior describing circles obviously depend upon the diameter of the intermediate circle, and as this may, as already stated, be selected, within certain limits, at will, it is evident that the relative diameters of the interior and exterior describing circles will vary in proportion, the interior becoming smaller and the exterior larger, while from the very mode of construction the radius of the two will equal that of the axes of the wheel and pinion. Thus in Fig. 69 the radii of S, T, equal A B, or the line of centres, and their diameters, there-

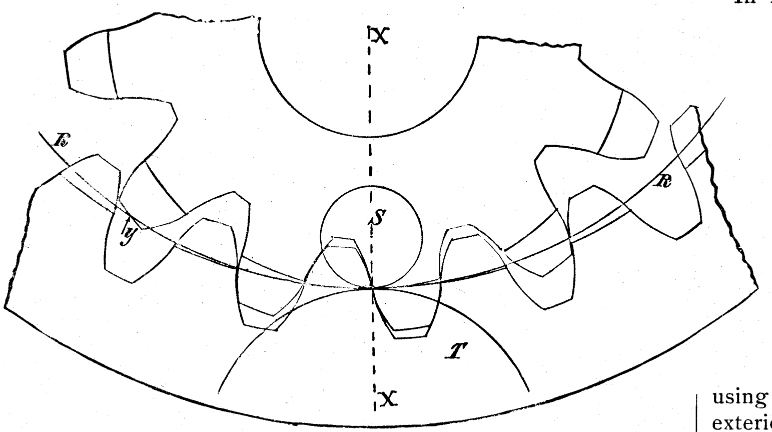

Fig. 70.

fore, equal the radius of the annular wheel, as is shown by dotting them in at the upper half of the figure. But after their diameters have been determined by this construction either of them may be decreased in diameter and the teeth of the wheels will clear (and not interfere as in Fig. 62), but the action will be the same

having 22 and the pinion 12 teeth), the diameter of the intermediate circle having been enlarged to decrease the diameter of S and increase that of T, and as these are left of the diameter derived from the construction there is receding action along R from the line of centres to T.

In Fig. 71 are represented a wheel and pinion, the pinion having but four teeth less than the wheel, and a tooth, J, being shown in position in which it has contact at two places. Thus at *k* it is in contact with the flank of a tooth on the annular wheel, while at L it is in contact with the face of the same tooth.

As the faces of the teeth on the wheel do not have contact higher than point *t*, it is obvious that instead of having them $\frac{3}{10}$ of the pitch as at the bottom of the figure, we may cut off the portion X without diminishing the arc of contact, leaving them formed as at the top of the figure. These faces being thus reduced in height we may correspondingly reduce the depth of flank on the pinion by filling in the portion G, leaving the teeth formed as at the top of the pinion. The teeth faces of the wheel being thus reduced we may, by using a sufficiently large intermediate circle, obtain interior and exterior describing circles that will form teeth that will permit of the pinion having but one tooth less than the wheel, or that will form a wheel having but one tooth more than the pinion.

The limits to the diameter of the intermediate describing circle are as follows: in Fig. 72 it is made equal in diameter to the pitch diameter of the pinion, hence B will represent the centre of the intermediate circle as well as of the pinion, and the pitch circle of the pinion will also represent the intermediate circle R. To

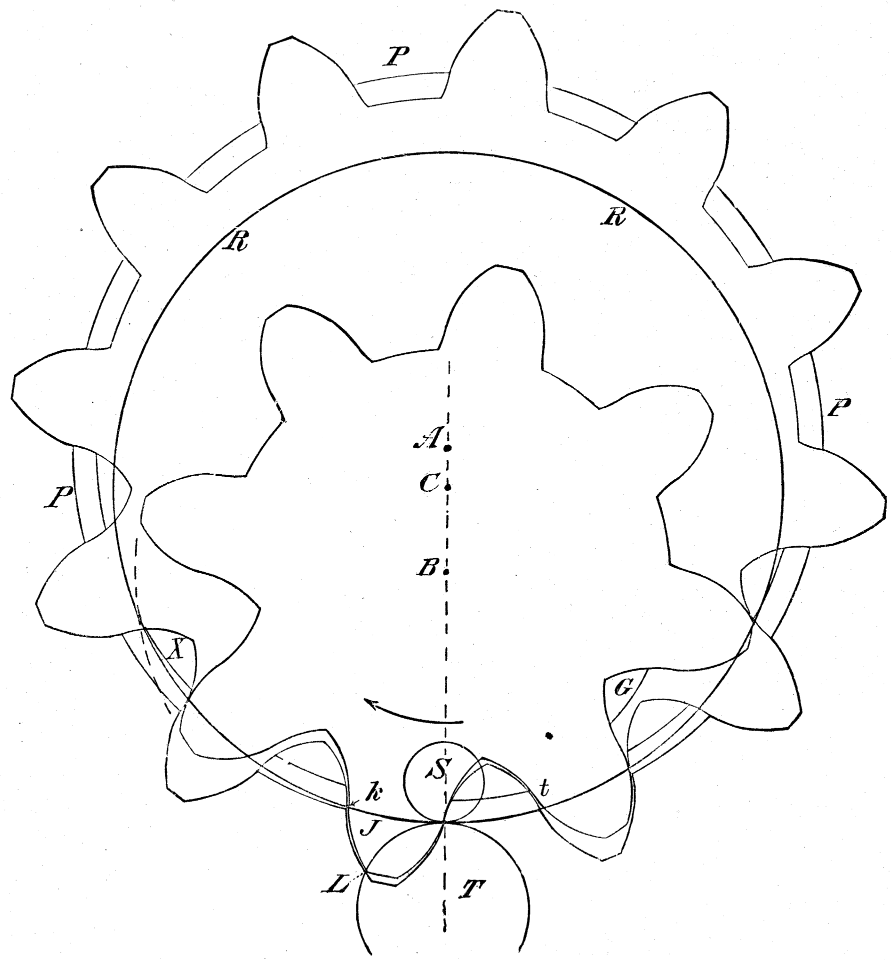

Fig. 71.

as in ordinary gear, or in other words there will be no arc of action on the circle R. But S cannot be increased without correspondingly decreasing T, nor can T be increased without correspondingly decreasing S.

Fig. 70 shows the same pair of gears as in Fig. 68 (the wheel

obtain the radius for the interior describing circle we subtract the radius of the intermediate circle from the radius of the annular wheel, which gives A P, hence the pitch circle of the pinion also represents the interior circle R. But when we come to obtain the radius for the exterior describing circle (T), by subtracting the radius of the pinion from that of the intermediate circle, we find that the two being equal give O for the radius of (T), hence there could be no flanks on the pinion.

Now suppose that the intermediate circle be made equal in diameter to the pitch circle of the annular wheel, and we may

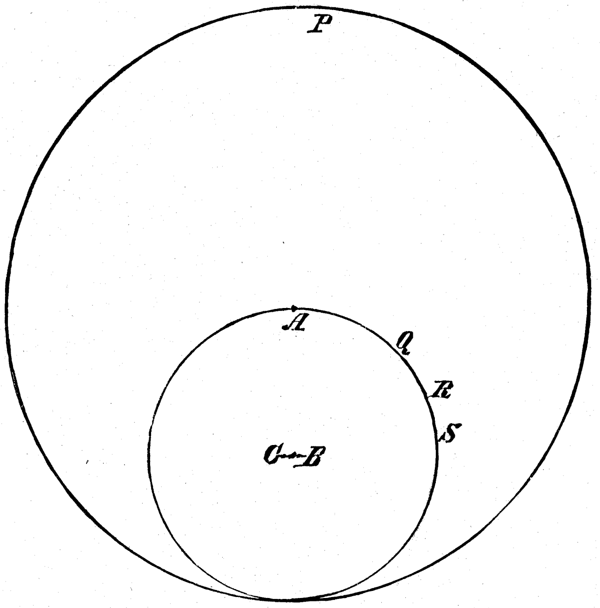

Fig. 72.

obtain the radius for the exterior describing circle T; by subtracting the radius of the pinion from that of the intermediate circle, we shall obtain the radius A B; hence the radius of (T) will equal that of the pinion. But when we come to obtain the radius for the interior describing circle by subtracting the radius of the intermediate circle from that of the annular wheel, we find these two to be equal, hence there would be no interior describing circle, and, therefore, no faces to the pinion.

The action of the teeth in internal wheels is less a sliding and more a rolling one than that in any other form of toothed gearing. This may be shown as follows: In Fig. 73 let A A represent the pitch circle of an external pinion, and B B that of an internal one, and P P the pitch circle of an external wheel for A A or an internal one for B B, the point of contact at the line of centres being at C, and the direction of rotation P P being as denoted by the arrow; the two pinions being driven, we suppose a point at C, on the pitch circle P P, to be coincident with a point on each of the two pinions at the line of centres. If P P be rotated so as to bring this point to the position denoted by D, the point on the external pinion having moved to E, while that on the internal

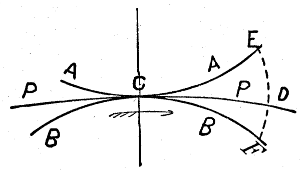

Fig. 73.

pinion has moved to F, both having moved through an arc equal to C D, then the distance from E to D being greater than from D to F, more sliding motion must have accompanied the contact of the teeth at the point E than at the point F; and the difference in the length of the arc E D and that of F D, may be taken to represent the excess of sliding action for the teeth on E; for whatever, under any given condition, the amount of sliding contact may be, it will be in the proportion of the length of E D to that of F D. Presuming, then, that the amount of power transmitted be equal for the two pinions, and the friction of all other things being equal—being in proportion to the space passed (or in this case slid) over—it is obvious that the internal pinion has the least friction.