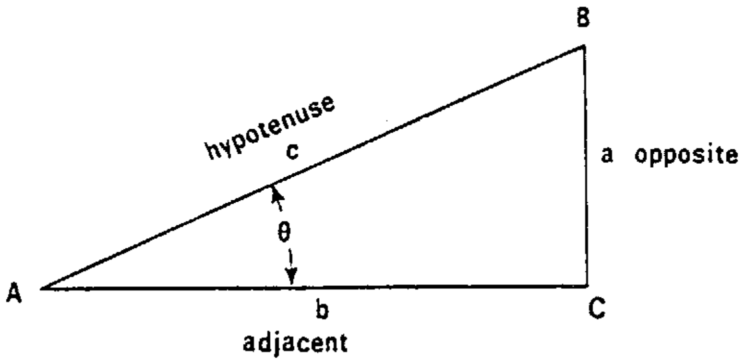
APPENDIX C

INTRODUCTION TO TRIGONOMETRY

Trigonometry is that branch of mathematics which deals with the relationships between the angles and sides of triangles. There are six such basic relationships which are called *functions*. They are: sine (sin), cosine (cos), tangent (tan), cotangent (cot), secant (sec), and cosecant (csc).



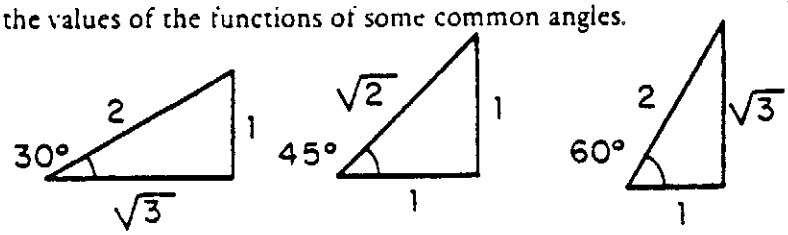
In a right triangle ABC, with a, b, and c being the sides opposite the corresponding angles, the trigonometric functions are expressed as the following ratios:

$$\text{sine } A = \sin A = \frac{a}{c} = \frac{opp}{hyp} \qquad \text{cosine } A = \cos A = \frac{b}{c} = \frac{adj}{hyp}$$

$$\text{tangent } A = \tan A = \frac{a}{b} = \frac{opp}{adj} \qquad \text{cotangent } A = \cot A = \frac{b}{a} = \frac{adj}{opp}$$

$$\text{secant } A = \sec A = \frac{c}{b} = \frac{hyp}{adj} \qquad \text{cosecant } A = \csc A = \frac{c}{a} = \frac{hyp}{opp}$$

The table below shows the values of the functions of some common angles.



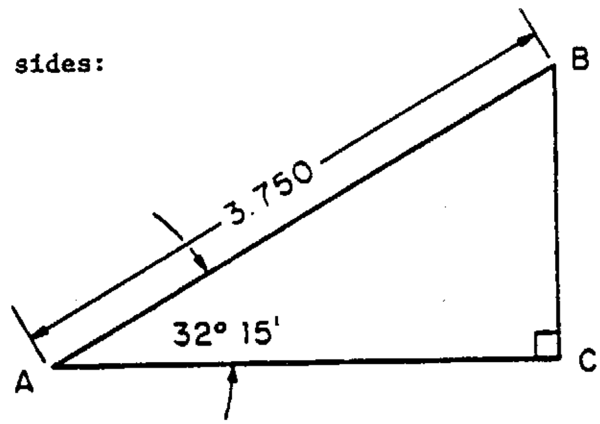
	30°	45°	60°
sin	1/2 (.5)	$1/2\sqrt{2}$ (.707107)	$1/2\sqrt{3}$ (.866025)
cos	$1/2\sqrt{3}$ (.866025)	$1/2\sqrt{2}$ (.707107)	1/2 (.5)
tan	$1/3\sqrt{3}$ (.57735)	1.	$\sqrt{3}$ (1.73205)
cot	$\sqrt{3}$ (1.73205)	1.	$1/3\sqrt{3}$ (.57735)
sec	$\frac{2}{\sqrt{3}}$ (1.15470)	$\frac{2}{\sqrt{2}}$ (1.414214)	2.
csc	2.	$\frac{2}{\sqrt{2}}$ (1.414214)	$\frac{2}{\sqrt{3}}$ (1.15470)

SOLVING PROBLEMS IN TRIGONOMETRY

With a basic knowledge of the use of trigonometry, problems can now be solved for unknown sides and angles of right triangles when one angle and one side are known, or when two sides are known, by applying the appropriate function and solving for the unknown element.

Problem 1

Find the unknown sides:



$$\sin A = \sin 32° 15' = .53361$$

$$\sin A = \frac{BC}{AB}$$

Substitute the known values and solve for BC:

$$.53361 = \frac{BC}{3.750}$$

$$BC = 2.0010$$

Likewise, to solve for AC,

$$\cos A = \cos 32° 15' = .84573$$

$$\cos A = \frac{AC}{AB}$$

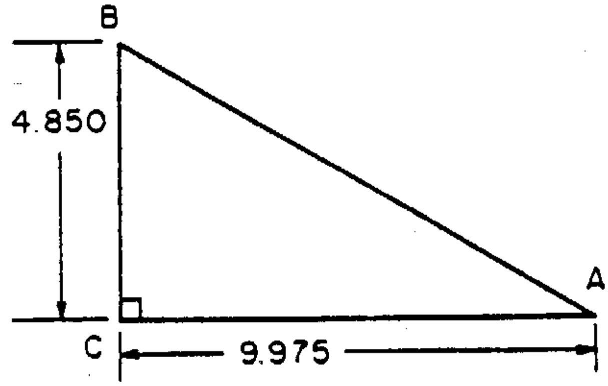
$$.84573 = \frac{AC}{3.750}$$

$$AC = 3.171$$

Problem 2

Angle C = 90°

Find angle A, angle B, side AB.



$$\tan A = \frac{BC}{AC} = \frac{4.850}{9.975} = .48622$$

Find the closest tangent values to .48622.

tan 25° 56' 00" = .48629
tan 25° 55' X" = .48622
tan 25° 55' 00" = .48593

Interpolate by setting up the following proportion and solving for x.

$$\frac{x}{60} = \frac{(.48622 - .48593)}{(.48629 - .48593)}$$

$$\frac{x}{60} = \frac{.00029}{.00036}$$

x = 48"

angle A = 25° 55' 48"

angle B = 180° - 90° - 25° 55' 48"

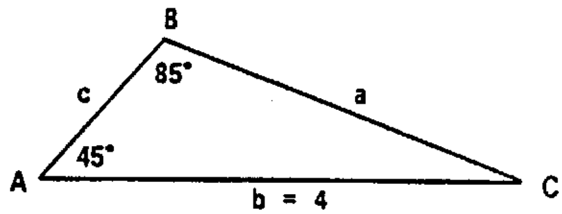
angle B = 179° 59' 60" - 115° 55' 48"

angle B = 64° 4' 12"

LAW OF SINES

The length of the sides of a triangle are proportional to the sines of the angles opposite them:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Problem 3

Given: $\angle A = 45°$, $\angle B = 85°$, $b = 4$

Find: $\angle C$, side a, c

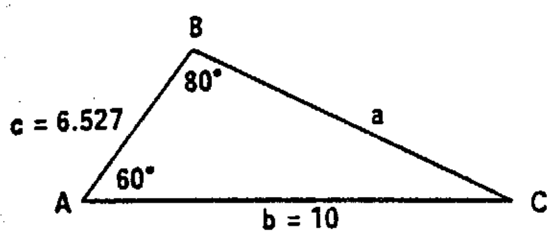
Solution	Explanation
$\angle C = 180° - (\angle A + \angle B)$	1) A triangle contains a maximum of 180°
$\angle C = 180° - (45° + 85°)$	2) Substitution
$\angle C = 180° - 130°$	3) Subtraction
$\angle C = 50°$	4) Answer

Using the Law of Sines

Explanation		
Law of sine	$\frac{b}{\sin B} = \frac{a}{\sin A}$	$\frac{b}{\sin B} = \frac{c}{\sin C}$
Substitution	$\frac{4}{\sin 85°} = \frac{a}{\sin 45°}$	$\frac{4}{\sin 85°} = \frac{c}{\sin 50°}$
Substitution from natural trig functions of angles	$\frac{4}{.99619} = \frac{a}{.70711}$	$\frac{4}{.99619} = \frac{c}{.76604}$
Cross multiplication	$.99619a = 2.82844$	$.99619c = 3.06416$
Division of both sides by .99619	$a = \frac{2.82844}{.99619}$	$c = \frac{3.06416}{.99619}$
Answer	$a = 2.839$	$c = 3.0758$

LAW OF COSINES

The square of the length of a side of a triangle equals the sum of the squares of the lengths of the other two sides minus twice the product of these two sides times the cosine of the angle between them.



$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$b^2 = a^2 + c^2 - 2ac(\cos B)$$
$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

Problem 4

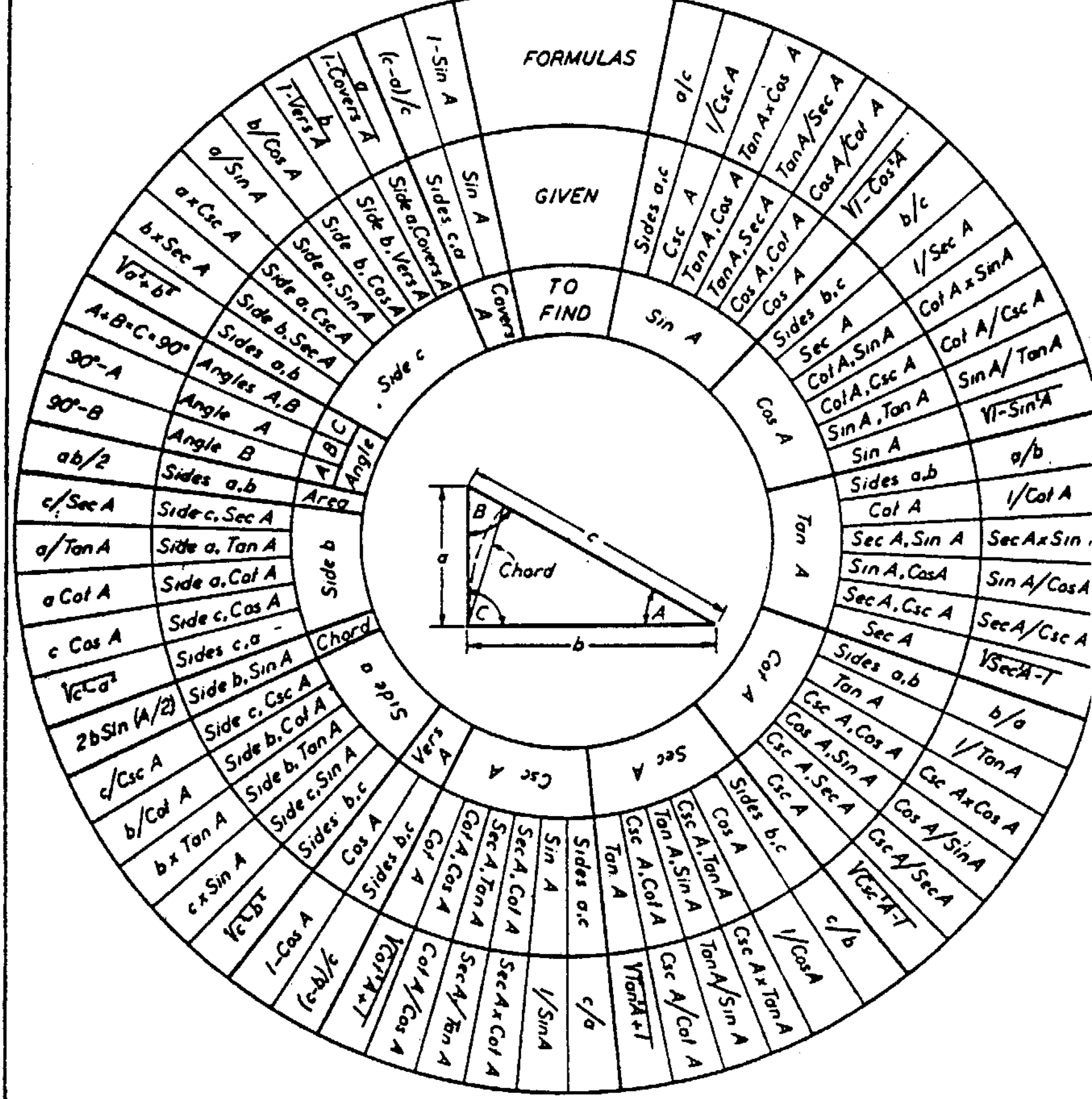
Given: Triangle ABC

$\angle A = 60°, \ \angle B = 80°, \ b = 10, \ c = 6.527$

Find: $\angle C$, side a

Solution	Explanation
$\angle C = 180° - (\angle A + \angle B)$	1) A triangle contains a maximum of 180°
$\angle C = 180° - (60° + 80°)$	2) Substitution
$\angle C = 180° - 140°$	3) Subtraction
$\angle C = 40°$	4) Answer
$a^2 = b^2 + c^2 - 2bc \cos A$	1) Law of cosines
$a^2 = (10)^2 + (6.527)^2 - 2(10)(6.527)(.5)$	2) Substitution
$a^2 = 100 + 42.60 - 2(65.27)(.5)$	3) Collecting terms
$a^2 = 100 + 42.60 - (130.54)(.5)$	4) Collecting terms
$a^2 = 142.60 - 65.27$	5) Substitution
$a^2 = 77.33$	6) Remainder
$a = \sqrt{77.33}$	7) Determining Square Root
$a = 8.7937$ or 8.794	8) Answer

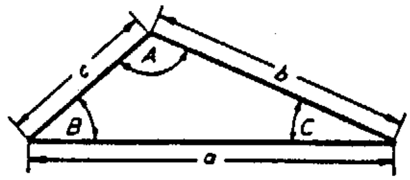
TRIGONOMETRIC CALCULATIONS - I



For handy reference, functional relationships and identities for right triangles are arranged in this chart. In using it, draw a triangle approximately to scale, label it with the same letters shown here, and designate all the known parts. Functional relationships and identities for oblique triangles are charted on the following page.

TRIGONOMETRIC CALCULATIONS - II

To find	Given	Formulas
A	B,C	$180° - (B+C)$
Tan A	a,b,C	$\frac{a \times \sin C}{b-(a \times \cos C)}$
Cos A	a,b,c	$\frac{b^2+c^2-a^2}{2bc}$
Sin A	a,c,C	$\frac{a \times \sin C}{c}$
Sin A	a,b,B	$\frac{a \times \sin B}{b}$
Tan A	a,c,B	$\frac{a \times \sin B}{c-(a \times \cos B)}$
B	A,C	$180° - (A+C)$
Sin B	a,b,A	$\frac{b \times \sin A}{a}$
Cos B	a,b,c	$\frac{c^2+a^2-b^2}{2ac}$
Tan B	b,c,A	$\frac{b \times \sin A}{c-(b \times \cos A)}$
Sin B	b,c,C	$\frac{b \times \sin C}{c}$



To find	Given	FORMULAS		Given	To find
a	c,A,C	$\frac{c \times \sin A}{\sin C}$	$\frac{a \times \sin B}{\sin A}$	a,A,B	b
a	b,A,B	$\frac{b \times \sin A}{\sin B}$	$\frac{c \times \sin B}{\sin C}$	c,B,C	b
a	b,c,A	$\sqrt{b^2+c^2-(2bc \times \cos A)}$	$\sqrt{c^2+a^2-(2ac \times \cos B)}$	a,c,B	b

Formulas	Given	To find
$180° - (A+B)$	A,B	C
$\frac{c \times \sin A}{a}$	a,c,A	Sin C
$\frac{c \times \sin A}{b-(c \times \cos A)}$	b,c,A	Tan C
$\frac{c \times \sin B}{b}$	b,c,B	Sin C
$\frac{c \times \sin B}{a-(c \times \cos B)}$	a,c,B	Tan C
$\frac{a^2+b^2-c^2}{2ab}$	a,b,c	Cos C
$\frac{a \times \sin C}{\sin A}$	a,A,C	c
$\sqrt{a^2+b^2-(2ab \times \cos C)}$	a,b,C	c
$\frac{b \times \sin C}{\sin B}$	b,B,C	c
$\frac{ab \times \sin C}{2}$	a,b,C	Area
$\sqrt{S(S-a)(S-b)(S-c)}$	a,b,c	Area $S=\frac{1}{2}(a+b+c)$

Arranged in this chart are the functional relationships and identities for oblique triangles; those for right triangles are shown in the chart on the preceding page. Use of the charts is best effected by drawing a sketch of the triangles approximately to scale, letter them as shown in the charts, and designate all known parts.